

$$\ln[r_] := ff[r_] = 1 - r^{-3};$$

$$\text{EquB} = 2 * \left( \frac{(I * w - r) * (I * w * r - q^2)}{r^2 * f[r]} + I * w - \frac{q^2}{3 * r} \right) * hvv[r] +$$

$$\left( w^2 - q^2 + q^2 * \frac{f[r]}{3} \right) * r^3 * f[r] * hrr[r] - I * w * \left( \frac{3}{f[r]} * \left( 1 + \frac{w^2}{r^2} \right) - 2 * \left( 2 + \frac{q^2}{r^2} \right) + f[r] \right) * hL[r];$$

$$\text{EquC} = hvv'[r] - \left( 1 - \frac{q^2}{I * w} * \frac{1}{r} \right) * \frac{hvv[r]}{r} -$$

$$\frac{3}{2} * \left( \frac{I * w}{2} + \frac{q^2}{I * w} * \frac{f[r]}{3} \right) * r^2 * f[r] * hrr[r] - \frac{3}{2 * r^4} * hL[r];$$

$$\text{EquD} =$$

$$hrr'[r] + 3 * \left( 2 - \frac{I * w}{r} + \left( \frac{q^2 + I * w * r}{r * (r - I * w)} - 2 \right) * \frac{f[r]}{3} - \frac{q^2}{I * w * (r - I * w)} * \left( \frac{f[r]}{3} \right)^2 \right) * \frac{hrr[r]}{r * f[r]} -$$

$$\frac{1 + 2 * q^2 * r + 3 * I * w * r^2}{r - I * w} * \frac{hL[r]}{r^7 * f[r]} + \frac{2 * (q^2 - 3 * I * w * r)}{3 * I * w * (r - I * w)} * \frac{hvv[r]}{r^5 * f[r]};$$

$$\text{EquE} = hL'[r] - \left( 1 + \frac{I * w}{r} + \frac{f[r]}{3} \right) * \frac{3}{2 * f[r]} * \frac{hL[r]}{r} -$$

$$\left( 1 - \frac{q^2}{I * w} * \frac{1}{r} \right) * \frac{1}{f[r]} * \frac{hvv[r]}{r} - \frac{q^2}{2 * I * w} * r^2 * f[r] * hrr[r];$$

$$\text{jieS} = \text{Simplify[}$$

$$\text{Solve[ \{ EquB == 0, EquC == 0, EquD == 0, EquE == 0 \}, \{ hvv'[r], hL'[r], hL[r], hvv[r] \} ]]$$

$$\text{hvp}[r_] = hvv'[r] /. \text{jieS}[[1]];$$

$$\text{hLp}[r_] = hL'[r] /. \text{jieS}[[1]];$$

$$\text{hL0}[r_] = hL[r] /. \text{jieS}[[1]];$$

$$\text{hvv0}[r_] = hvv[r] /. \text{jieS}[[1]];$$

$$\begin{aligned}
\text{Out[*]} = & \left\{ \left\{ \text{hvv}'[r] \rightarrow (-r (54 (r - i w)^2 (q^2 - i r w) (-2 r + 2 r^4 + i w + i r^3 w + r^2 w^2) + \right. \right. \\
& 9 (r - i w) (q^4 (-2 - 6 r^3 + 4 i r^2 w) + q^2 r (8 r - 20 r^4 - 7 i w + 17 i r^3 w) + \\
& r^2 w (-16 i r + 20 i r^4 - 13 w + 13 r^3 w + 6 i r^2 w^2)) f[r] + \\
& 3 r (q^4 (2 - 2 i r^2 w) + r^2 w (12 i r - 28 i r^4 + 17 w - 54 r^3 w + 21 i r^2 w^2) + \\
& q^2 (24 r^5 + i r w - 46 i r^4 w - 21 r^3 w^2)) f[r]^2 + \\
& r^3 (6 q^4 r + 3 r^3 w (4 i r + 9 w) + q^2 (-4 + 4 r^3 + 3 i r^2 w)) f[r]^3 - 4 q^2 r^6 f[r]^4 \text{hrr}[r] + \\
& 6 r^3 (i r + w) f[r] (3 (i q^2 + r w) (-r + r^4 + i w + r^2 w^2) - \\
& r (2 i q^4 r + r (-3 + 4 r^3) w + q^2 (-i + 4 i r^3 + 2 r^2 w)) f[r] + \\
& r^4 (i q^2 + r w) f[r]^2) \text{hrr}'[r]) / \\
& (4 (6 q^4 r (r - i w) - 3 q^2 (-1 + r^3) (r - i w) + 3 r (-1 + 3 r^3) w (i r + w) + \\
& r (q^2 - 3 i r w) (-1 + 4 r^3 - 3 i r^2 w) f[r] + r^4 (-q^2 + 3 i r w) f[r]^2)), \\
\text{hL}'[r] \rightarrow & (-r^2 (-18 r (q^4 + 2 q^2 r (r - i w) + r^2 (-i r - 2 w) w) (2 r^2 - 3 i r w - w^2) - \\
& 3 (q^4 (-1 + r^3) - 4 q^2 r^3 (4 r^2 - 7 i r w - 3 w^2) + r^2 w (16 i r^4 - w + 35 r^3 w - 18 i r^2 w^2)) f[r] + \\
& r^2 (3 q^4 r + 2 q^2 (-1 + r^3) + 3 r^3 w (4 i r + 5 w)) f[r]^2 - 2 q^2 r^5 f[r]^3) \text{hrr}[r] + 2 r^5 (r - i w) \\
& f[r] (3 (q^4 + 2 q^2 r (r - i w) + r^2 (-i r - 2 w) w) + r^2 (-2 q^2 + 3 i r w) f[r]) \text{hrr}'[r]) / \\
& (2 (6 q^4 r (r - i w) - 3 q^2 (-1 + r^3) (r - i w) + 3 r (-1 + 3 r^3) w (i r + w) + \\
& r (q^2 - 3 i r w) (-1 + 4 r^3 - 3 i r^2 w) f[r] + r^4 (-q^2 + 3 i r w) f[r]^2)), \\
\text{hL}[r] \rightarrow & (r^5 (r - i w) (3 (-3 i (q^2 - i r w) (2 r^2 - 3 i r w - w^2) + \\
& (-i q^4 + 2 r^2 (4 r - 3 i w) w + q^2 r (4 i r + 3 w)) f[r] - 2 r^3 w f[r]^2) \text{hrr}[r] + \\
& r^2 f[r] (-3 (r - i w) (i q^2 + r w) + r (i q^2 + 3 r w) f[r]) \text{hrr}'[r])) / \\
& (-6 i q^4 r (r - i w) + 3 r (-1 + 3 r^3) (r - i w) w + 3 q^2 (-1 + r^3) (i r + w) - \\
& r (i q^2 + 3 r w) (-1 + 4 r^3 - 3 i r^2 w) f[r] + r^4 (i q^2 + 3 r w) f[r]^2), \text{hvv}[r] \rightarrow \\
& (r^5 (27 i (r - i w)^2 w (2 r^2 + i r w + w^2) - 9 i (r - i w) w (3 q^2 (r - i w) + r (10 r^2 - 5 i r w + 3 w^2)) \\
& f[r] + 3 (2 q^4 (r - i w) + q^2 (1 - r^3 + 3 i r^2 w + 3 r w^2) + w (14 i r^4 - w + 21 r^3 w - 6 i r^2 w^2)) \\
& f[r]^2 + (q^2 (-1 + 4 r^3) + 3 r^3 (-2 i r - 3 w) w) f[r]^3 - q^2 r^3 f[r]^4) \text{hrr}[r] + \\
& 3 r^7 w (i r + w) f[r] (3 (r^2 + w^2) - 2 (q^2 + 2 r^2) f[r] + r^2 f[r]^2) \text{hrr}'[r]) / \\
& (2 (6 q^4 r (r - i w) - 3 q^2 (-1 + r^3) (r - i w) + 3 r (-1 + 3 r^3) w (i r + w) + \\
& r (q^2 - 3 i r w) (-1 + 4 r^3 - 3 i r^2 w) f[r] + r^4 (-q^2 + 3 i r w) f[r]^2)) \} \}
\end{aligned}$$

`In[*]:= Equrr0A = Simplify[hLp[r] - hL0'[r]];`

`EqurrA = Collect[ $\frac{\text{Equrr0A}}{\text{Coefficient}[\text{Equrr0A}, \text{hrr}'[r]]}$ , {hrr''[r], hrr'[r], hrr[r]}, Simplify]`

`Equrr0 = Simplify[hvvp[r] - hvv0'[r]];`

`Equrr = Collect[ $\frac{\text{Equrr0}}{\text{Coefficient}[\text{Equrr0}, \text{hrr}'[r]]}$ , {hrr''[r], hrr'[r], hrr[r]}, Simplify];`

`Simplify[EqurrA - Equrr == 0 /. f -> ff]`

$$\begin{aligned}
\text{Out[*]} = & (\text{hrr}[r] (r^3 (q^6 (6 r + 9 r^4 - 9 i r^3 w) + 9 r^4 w^2 (-24 r + 96 r^4 + 18 i w - 125 i r^3 w - 39 r^2 w^2) + \\
& 3 q^2 r w (2 i - 40 i r^3 + 166 i r^6 - 30 r^2 w + 193 r^5 w - 45 i r^4 w^2) + \\
& q^4 (-2 + 16 r^3 - 62 r^6 - 30 i r^2 w + 81 i r^5 w + 27 r^4 w^2)) f[r]^3 + \\
& r^6 (3 q^6 r + 9 r^4 (-12 r + 7 i w) w^2 + 3 q^2 r w (4 i - 26 i r^3 - 17 r^2 w) + q^4 (-4 + 10 r^3 - 15 i r^2 w))
\end{aligned}$$

$$\begin{aligned}
& f[r]^4 - 2q^2 r^9 (q^2 - 3irw) f[r]^5 - 9(r-iw) f[r] \\
& (2q^8 (r+7r^4 - 8ir^3w) + q^6 (1+12r^3 - 83r^6 - 12ir^2w + 124ir^5w + 42r^4w^2) + \\
& r^3w^2 (168r^4 - 288r^7 - iw - 212ir^3w + 387ir^6w - 66r^2w^2 + 120r^5w^2) + \\
& q^4 (48r^8 - irw + 68ir^4w + 177ir^7w + 16r^3w^2 + 342r^6w^2 + r^5 (-60 - 126iw^3)) + \\
& q^2 r^2 w (-200ir^7 + w + 296r^3w - 241r^6w - 96ir^2w^2 + 48ir^5w^2 - 4r^4 (-55i + 3w^3)) + \\
& 4r^6 (ir+w) (q^4 r (-2ir-5w) + 2q^2 w (-1-3r^3 + 2ir^2w) + rw^2 (2i + 3r^2w)) f'[r] - \\
& 3rf[r]^2 (6q^8 r^3 (r-iw) + q^6 (-1-6r^3 + 25r^6 + 6ir^2w - 42ir^5w - 18r^4w^2) - \\
& q^4 r (2r+90r^7 - 5iw + 54ir^3w + 23ir^6w + 12r^2w^2 + 270r^5w^2 + r^4 (-44 - 162iw^3)) + \\
& 3r^3w^2 (264r^7 + iw + 176ir^3w - 505ir^6w + 62r^2w^2 - 294r^5w^2 + 24ir^4 (5i + 2w^3)) + q^2 r^2 \\
& w (2ir + 490ir^7 + w - 456r^3w + 869r^6w + 168ir^2w^2 - 444ir^5w^2 - 6r^4 (50i + 9w^3)) + \\
& 2r^6 (q^6 (r-iw) - 4q^4 r^2 (r-iw) + 6r (r-iw) w^2 + \\
& q^2 w (2ir + 12ir^4 + 2w + 21r^3w - 9ir^2w^2)) f'[r] - \\
& 18r (r-iw)^2 (-3 (2q^8 r (2r-iw) + r^3w^2 (12r^2 - 18r^5 - 13irw + 12ir^4w - 3w^2) + \\
& q^6 (2r - 18r^4 - iw + 19ir^3w + 4r^2w^2) - iq^2 r^2 w (-22r^2 + 14r^5 + 25irw - 7ir^4w + 6 \\
& w^2 + 2r^3w^2) + q^4 r (4r^5 + 10irw + 27ir^4w + 2w^2 + 29r^3w^2 + r^2 (-10 - 6iw^3))) + \\
& r^2 (2q^8 r + q^6 (1-9r^3 + 6ir^2w) + r^3w^2 (2r-3iw - 12ir^3w - 9r^2w^2) + \\
& q^2 r^2 w (4ir + 12ir^4 + 5w + 25r^3w - 12ir^2w^2) + \\
& q^4 r (-2r - 4r^4 + iw + 26ir^3w + 15r^2w^2)) f'[r] ) / \\
& (2r^5 (r-iw) f[r] (-3 (r-iw) (iq^2 + rw) + r (iq^2 + 3rw) f[r]) \\
& (-6iq^4 r (r-iw) + \\
& 3r (-1+3r^3) (r-iw) w + 3q^2 (-1+r^3) (ir+w) - \\
& r (iq^2 + 3rw) (-1+4r^3 - 3ir^2w) f[r] + \\
& r^4 (iq^2 + 3rw) f[r]^2) + \\
& ((r^3 (q^4 (-6r+9r^4 + 5iw - 11ir^3w - 3r^2w^2) + 3q^2 rw (11ir - 11ir^4 + 9w - 11r^3w + 2ir^2w^2) + \\
& 9r^2w^2 (5r - 4r^4 - 4iw + 4ir^3w + r^2w^2)) f[r]^3 + \\
& r^6 (q^4 (-3r+2iw) + 9r^2 (2r-iw) w^2 + 3q^2 rw (5ir+3w)) f[r]^4 - \\
& 9 (r-iw)^3 (2q^6 r + 2iq^2 r (-1+2r^3) w + r^2 (-1+3r^3) w^2 - q^4 (-1+r^3 + 2ir^2w)) \\
& (6r - 3iw + r^2 f'[r]) + \\
& 3r (r-iw)^2 f[r] (-3 (2q^6 r (r-iw) + q^4 (9r^4 - iw - 9ir^3w - r^2w^2) + q^2 rw (7ir - 41 \\
& ir^4 + 3w - 49r^3w + 12ir^2w^2) + r^2w^2 (7r - 36r^4 - 4iw + 42ir^3w + 9r^2w^2)) + \\
& 2r^2 (2q^6 r + 2iq^2 r (-2+3r^3) w + 3r^2 (-1+3r^3) w^2 - q^4 (-1+r^3 + 6ir^2w)) f'[r] - \\
& r^2 (r-iw) f[r]^2 (-3 (10q^6 r (r-iw) + q^4 (8r+7r^4 - 7iw - 44ir^3w - 36r^2w^2) + \\
& q^2 rw (-26ir - 45ir^4 - 22w - 88r^3w + 39ir^2w^2) + \\
& 3r^2w^2 (-4r - 16r^4 + 3iw + 32ir^3w + 15r^2w^2)) + \\
& r^2 (-q^4 (-1+r^3) + 9r^2w^2 (-1+3r^3 - 2ir^2w) + 6q^2 rw (-i + 2ir^3 + r^2w)) f'[r] ) \\
& hrr'[r] / (r^2 (r-iw) f[r] (-3 (r-iw) (iq^2 + rw) + r (iq^2 + 3rw) f[r]) \\
& (-6iq^4 r (r-iw) + 3r (-1+3r^3) (r-iw) w + \\
& 3q^2 (-1+r^3) (ir+w) - r (iq^2 + 3rw) (-1+4r^3 - 3ir^2w) f[r] + \\
& r^4 (iq^2 + 3rw) f[r]^2) + hrr''[r]
\end{aligned}$$

`Out[ ]= True`

`In[ ]:= Equrr18 =`

$$\text{hrr}''[r] + \left( \frac{2}{r} + \frac{f'[r]}{f[r]} - 3 * \frac{I * w - 2 * r}{r^2 * f[r]} \right) * \text{hrr}'[r] + \frac{3}{r^2 * f[r]} * \left( 4 - 3 * \frac{I * w}{r} - \frac{q^2}{r^2} \right) * \text{hrr}[r];$$

`Simplify[Equrr18 - Equrr == 0 /. f -> ff]`

`Out[ ]= True`