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ClearAll[phvv, hvv, hrr, hL, phL, xphL, xhL, Cof3hvv,
Cof3hrr, Cof3hL, Cof2hL, Cof2hvv, Cof2hrr, Cof1hrr, Cof1hL, Cof1hvv]
f[r_] = 1 -  $\frac{1}{r^3}$ ;

Cof3hvv[r_] = 2  $\left( \frac{(I \omega - r) (I \omega r - q^2)}{r^2 f[r]} + I \omega - \frac{q^2}{3 r} \right)$ ;

Cof3hrr[r_] =  $\left( \omega^2 - q^2 + q^2 \frac{f[r]}{3} \right) r^3 f[r]$ ;

Cof3hL[r_] = -I  $\omega \left( \frac{3}{f[r]} \left( 1 + \frac{\omega^2}{r^2} \right) - 2 \left( 2 + \frac{q^2}{r^2} \right) + f[r] \right)$ ;

Eqhvv = Cof3hvv[r]  $\times$  phvv[r] + Cof3hrr[r]  $\times$  hrr[r] + Cof3hL[r]  $\times$  hL[r];
Print["Equation for hvv: ", Eqhvv]
hvv[r_] = phvv[r] /. Solve[Eqhvv == 0, phvv[r]] [[1]];
Print["Obtained expression for hvv: ", hvv[r]]

Cof2hL[r_] = -  $\left( 1 + \frac{I \omega}{r} + \frac{f[r]}{3} \right) \frac{3}{2 f[r]} \frac{1}{r}$ ;

Cof2hvv[r_] = -  $\left( 1 - \frac{q^2}{I \omega} \frac{1}{r} \right) \frac{1}{f[r]} \frac{1}{r}$ ;

Cof2hrr[r_] = -  $\frac{q^2}{2 I \omega} r^2 f[r]$ ;

EqhL = phL[r] + Cof2hL[r]  $\times$  hL[r] + Cof2hvv[r]  $\times$  hvv[r] + Cof2hrr[r]  $\times$  hrr[r];
Print["Equation for hL[r]: ", EqhL]
xphL = Simplify[phL[r] /. Solve[EqhL == 0, phL[r]]] [[1]];
Print["Obtained first derivative of hL[r] ", xphL]

Cof1hrr[r_] = 3  $\left( 2 - \frac{I \omega}{r} + \left( \frac{q^2 + I \omega r}{r (r - I \omega)} - 2 \right) \frac{f[r]}{3} - \frac{q^2}{I \omega (r - I \omega)} \left( \frac{f[r]}{3} \right)^2 \right) \frac{1}{r f[r]}$ ;

Cof1hL[r_] = -  $\frac{1 + 2 q^2 r + 3 I \omega r^2}{r - I \omega} \frac{1}{r^7 f[r]}$ ;

Cof1hvv[r_] =  $\frac{2 (q^2 - 3 I \omega r)}{3 I \omega (r - I \omega)} \frac{1}{r^5 f[r]}$ ;

Eqhrr = D[hrr[r], r] + Cof1hrr[r]  $\times$  hrr[r] + Cof1hL[r]  $\times$  hL[r] + Cof1hvv[r]  $\times$  hvv[r];
Print["Equation for hrr[r]: ", Eqhrr]
xhL = Simplify[hL[r] /. Solve[Eqhrr == 0, hL[r]]] [[1]];
Print["Obtained hL[r]: ", xhL]
DEqhrr = D[Eqhrr, r];
Print["The first order derivative of the equation for hrr[r]: ", DEqhrr]
SDEqhrr = Simplify[{DEqhrr /. {hL'[r]  $\rightarrow$  xphL}} /. {hL[r]  $\rightarrow$  xhL}];
Print[
"The substituted equation for the first order derivative of the equation for hrr[r]: ",
SDEqhrr[r]]

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$$\text{SSDE} = \text{Simplify}\left[\frac{\text{SDEqhrr}}{\text{Coefficient}[\text{SDEqhrr}, \text{hrr}''[r]]}\right];$$

Print["The second order coefficient: ", Simplify[Coefficient[SSDE, hrr''[r]], " "]
Simplify[Coefficient[SSDE, hrr''[r]] - 1 == 0]

Print["The first order coefficient: ", Simplify[Coefficient[SSDE, hrr'[r]], " "]

$$\text{Simplify}\left[\text{Coefficient}[\text{SSDE}, \text{hrr}'[r]][[1]] - \left(\frac{2}{r} + \frac{D[f[r], r]}{f[r]} - 3 \frac{I \omega - 2 r}{r^2 f[r]}\right) == 0\right]$$

Print["The function expression: ", Simplify[Coefficient[SSDE, hrr[r]], " "]

$$\text{Simplify}\left[\text{Coefficient}[\text{SSDE}, \text{hrr}[r]][[1]] - \frac{3}{r^2 f[r]} \left(4 - 3 \frac{I \omega}{r} - \frac{q^2}{r^2}\right) == 0\right]$$

$$\text{Equation for hvv: } -i \omega \left(1 - 2 \left(2 + \frac{q^2}{r^2}\right) - \frac{1}{r^3} + \frac{3 \left(1 + \frac{\omega^2}{r^2}\right)}{1 - \frac{1}{r^3}}\right) \text{hL}[r] +$$

$$\left(1 - \frac{1}{r^3}\right) r^3 \left(-q^2 + \frac{1}{3} q^2 \left(1 - \frac{1}{r^3}\right) + \omega^2\right) \text{hrr}[r] + 2 \left(-\frac{q^2}{3 r} + i \omega + \frac{(-r + i \omega) (-q^2 + i r \omega)}{\left(1 - \frac{1}{r^3}\right) r^2}\right) \text{phvv}[r]$$

Obtained expression for hvv:

$$\left(\left(3 i \omega \text{hL}[r] + 6 i q^2 r \omega \text{hL}[r] + 6 i r^3 \omega \text{hL}[r] - 6 i q^2 r^4 \omega \text{hL}[r] + 9 i r^4 \omega^3 \text{hL}[r] + q^2 \text{hrr}[r] - 3 q^2 r^6 \text{hrr}[r] + 2 q^2 r^9 \text{hrr}[r] - 3 r^3 \omega^2 \text{hrr}[r] + 6 r^6 \omega^2 \text{hrr}[r] - 3 r^9 \omega^2 \text{hrr}[r]\right) / \left(2 r^2 \left(q^2 + 2 q^2 r^3 - 3 i r \omega - 3 i q^2 r^2 \omega - 3 r^3 \omega^2\right)\right)\right)$$

$$\text{Equation for hL}[r]: \frac{3 \left(-1 + \frac{1}{3} \left(-1 + \frac{1}{r^3}\right) - \frac{i \omega}{r}\right) \text{hL}[r]}{2 \left(1 - \frac{1}{r^3}\right) r} + \frac{i q^2 \left(1 - \frac{1}{r^3}\right) r^2 \text{hrr}[r]}{2 \omega} +$$

$$\left(\left(-1 - \frac{i q^2}{r \omega}\right) \left(3 i \omega \text{hL}[r] + 6 i q^2 r \omega \text{hL}[r] + 6 i r^3 \omega \text{hL}[r] - 6 i q^2 r^4 \omega \text{hL}[r] + 9 i r^4 \omega^3 \text{hL}[r] + q^2 \text{hrr}[r] - 3 q^2 r^6 \text{hrr}[r] + 2 q^2 r^9 \text{hrr}[r] - 3 r^3 \omega^2 \text{hrr}[r] + 6 r^6 \omega^2 \text{hrr}[r] - 3 r^9 \omega^2 \text{hrr}[r]\right) / \left(2 \left(1 - \frac{1}{r^3}\right) r^3 \left(q^2 + 2 q^2 r^3 - 3 i r \omega - 3 i q^2 r^2 \omega - 3 r^3 \omega^2\right)\right) + \text{phL}[r]\right)$$

Obtained first derivative of hL[r]

$$\left(\left(2 \left(3 q^4 r + q^2 \left(2 + 4 r^3 - 6 i r^2 \omega\right) - 3 r \omega \left(i + 2 r^2 \omega\right)\right) \text{hL}[r] + r \left(-1 + r^3\right) \left(-3 q^4 r + 2 q^2 \left(-1 + r^3\right) - 3 r^3 \omega^2\right) \text{hrr}[r]\right) / \left(2 r \left(q^2 \left(1 + 2 r^3 - 3 i r^2 \omega\right) - 3 r \omega \left(i + r^2 \omega\right)\right)\right)\right)$$

Equation for hrr[r]:

$$\frac{(1 + 2q^2 r + 3i r^2 \omega) hL[r]}{\left(1 - \frac{1}{r^3}\right) r^7 (r - i\omega)} + \frac{3 \left(2 + \frac{i q^2 \left(1 - \frac{1}{r^3}\right)^2}{9 (r - i\omega) \omega} - \frac{i\omega}{r} + \frac{1}{3} \left(1 - \frac{1}{r^3}\right) \left(-2 + \frac{q^2 + i r \omega}{r (r - i\omega)}\right)\right) hrr[r]}{\left(1 - \frac{1}{r^3}\right) r} -$$

$$\frac{(i (q^2 - 3i r \omega) (3i \omega hL[r] + 6i q^2 r \omega hL[r] + 6i r^3 \omega hL[r] - 6i q^2 r^4 \omega hL[r] + 9i r^4 \omega^3 hL[r] + q^2 hrr[r] - 3q^2 r^6 hrr[r] + 2q^2 r^9 hrr[r] - 3r^3 \omega^2 hrr[r] + 6r^6 \omega^2 hrr[r] - 3r^9 \omega^2 hrr[r]))}{\left(3 \left(1 - \frac{1}{r^3}\right) r^7 (r - i\omega) \omega (q^2 + 2q^2 r^3 - 3i r \omega - 3i q^2 r^2 \omega - 3r^3 \omega^2)\right) + hrr'[r]}$$

Obtained hL[r]: $\frac{1}{6 (q^4 + i\omega)}$

$$r (3 (q^4 (-1 + r^3) + q^2 r (4r + 2r^4 - 3i\omega - 6i r^3 \omega - 3r^2 \omega^2) + i\omega (-2 - 4r^3 + 6i r^2 \omega + 3i r^5 \omega + 3r^4 \omega^2)) hrr[r] + (-1 + r^3) (q^2 (1 + 2r^3 - 3i r^2 \omega) - 3r \omega (i + r^2 \omega)) hrr'[r])$$

The first order derivative of the equation for hrr[r]:

$$\frac{(2q^2 + 6i r \omega) hL[r]}{\left(1 - \frac{1}{r^3}\right) r^7 (r - i\omega)} + \frac{(1 + 2q^2 r + 3i r^2 \omega) hL[r]}{\left(1 - \frac{1}{r^3}\right) r^7 (r - i\omega)^2} + \frac{3 (1 + 2q^2 r + 3i r^2 \omega) hL[r]}{\left(1 - \frac{1}{r^3}\right)^2 r^{11} (r - i\omega)} +$$

$$\frac{7 (1 + 2q^2 r + 3i r^2 \omega) hL[r]}{\left(1 - \frac{1}{r^3}\right) r^8 (r - i\omega)} - \frac{9 \left(2 + \frac{i q^2 \left(1 - \frac{1}{r^3}\right)^2}{9 (r - i\omega) \omega} - \frac{i\omega}{r} + \frac{1}{3} \left(1 - \frac{1}{r^3}\right) \left(-2 + \frac{q^2 + i r \omega}{r (r - i\omega)}\right)\right) hrr[r]}{\left(1 - \frac{1}{r^3}\right)^2 r^5} -$$

$$\frac{3 \left(2 + \frac{i q^2 \left(1 - \frac{1}{r^3}\right)^2}{9 (r - i\omega) \omega} - \frac{i\omega}{r} + \frac{1}{3} \left(1 - \frac{1}{r^3}\right) \left(-2 + \frac{q^2 + i r \omega}{r (r - i\omega)}\right)\right) hrr[r]}{\left(1 - \frac{1}{r^3}\right) r^2} +$$

$$\frac{3 \left(-\frac{i q^2 \left(1 - \frac{1}{r^3}\right)^2}{9 (r - i\omega)^2 \omega} + \frac{2i q^2 \left(1 - \frac{1}{r^3}\right)}{3r^4 (r - i\omega) \omega} + \frac{i\omega}{r^2} + \frac{1}{3} \left(1 - \frac{1}{r^3}\right) \left(\frac{i\omega}{r (r - i\omega)} - \frac{q^2 + i r \omega}{r (r - i\omega)^2} - \frac{q^2 + i r \omega}{r^2 (r - i\omega)}\right) + \frac{-2 + \frac{q^2 + i r \omega}{r (r - i\omega)}}{r^4}\right) hrr[r]}{\left(1 - \frac{1}{r^3}\right) r} +$$

$$\frac{(i (q^2 - 3i r \omega) (6q^2 r^2 - 3i\omega - 6i q^2 r \omega - 9r^2 \omega^2) (3i \omega hL[r] + 6i q^2 r \omega hL[r] + 6i r^3 \omega hL[r] - 6i q^2 r^4 \omega hL[r] + 9i r^4 \omega^3 hL[r] + q^2 hrr[r] - 3q^2 r^6 hrr[r] + 2q^2 r^9 hrr[r] - 3r^3 \omega^2 hrr[r] + 6r^6 \omega^2 hrr[r] - 3r^9 \omega^2 hrr[r]))}{\left(3 \left(1 - \frac{1}{r^3}\right) r^7 (r - i\omega) \omega (q^2 + 2q^2 r^3 - 3i r \omega - 3i q^2 r^2 \omega - 3r^3 \omega^2)\right)^2} -$$

$$\frac{(3i \omega hL[r] + 6i q^2 r \omega hL[r] + 6i r^3 \omega hL[r] - 6i q^2 r^4 \omega hL[r] + 9i r^4 \omega^3 hL[r] + q^2 hrr[r] - 3q^2 r^6 hrr[r] + 2q^2 r^9 hrr[r] - 3r^3 \omega^2 hrr[r] + 6r^6 \omega^2 hrr[r] - 3r^9 \omega^2 hrr[r]))}{\left(\left(1 - \frac{1}{r^3}\right) r^7 (r - i\omega) (q^2 + 2q^2 r^3 - 3i r \omega - 3i q^2 r^2 \omega - 3r^3 \omega^2)\right) +$$

$$(i (q^2 - 3i r \omega) (3i \omega hL[r] + 6i q^2 r \omega hL[r] + 6i r^3 \omega hL[r] - 6i q^2 r^4 \omega hL[r] + 9i r^4 \omega^3 hL[r] + q^2 hrr[r] - 3q^2 r^6 hrr[r] + 2q^2 r^9 hrr[r] - 3r^3 \omega^2 hrr[r] + 6r^6 \omega^2 hrr[r] - 3r^9 \omega^2 hrr[r]))}{\left(3 \left(1 - \frac{1}{r^3}\right) r^7 (r - i\omega) \omega (q^2 + 2q^2 r^3 - 3i r \omega - 3i q^2 r^2 \omega - 3r^3 \omega^2)\right)^2} +$$

$$\begin{aligned}
& \left(3 \left(1 - \frac{1}{r^3} \right) r^7 (r - i \omega)^2 \omega (q^2 + 2 q^2 r^3 - 3 i r \omega - 3 i q^2 r^2 \omega - 3 r^3 \omega^2) \right) + \\
& (i (q^2 - 3 i r \omega) (3 i \omega hL[r] + 6 i q^2 r \omega hL[r] + 6 i r^3 \omega hL[r] - 6 i q^2 r^4 \omega hL[r] + 9 i r^4 \omega^3 hL[r] + \\
& \quad q^2 hrr[r] - 3 q^2 r^6 hrr[r] + 2 q^2 r^9 hrr[r] - 3 r^3 \omega^2 hrr[r] + 6 r^6 \omega^2 hrr[r] - 3 r^9 \omega^2 hrr[r])) / \\
& \left(\left(1 - \frac{1}{r^3} \right)^2 r^{11} (r - i \omega) \omega (q^2 + 2 q^2 r^3 - 3 i r \omega - 3 i q^2 r^2 \omega - 3 r^3 \omega^2) \right) + \\
& (7 i (q^2 - 3 i r \omega) (3 i \omega hL[r] + 6 i q^2 r \omega hL[r] + 6 i r^3 \omega hL[r] - 6 i q^2 r^4 \omega hL[r] + 9 i r^4 \omega^3 hL[r] + \\
& \quad q^2 hrr[r] - 3 q^2 r^6 hrr[r] + 2 q^2 r^9 hrr[r] - 3 r^3 \omega^2 hrr[r] + 6 r^6 \omega^2 hrr[r] - 3 r^9 \omega^2 hrr[r])) / \\
& \left(3 \left(1 - \frac{1}{r^3} \right) r^8 (r - i \omega) \omega (q^2 + 2 q^2 r^3 - 3 i r \omega - 3 i q^2 r^2 \omega - 3 r^3 \omega^2) \right) - \\
& \frac{(1 + 2 q^2 r + 3 i r^2 \omega) hL'[r]}{\left(1 - \frac{1}{r^3} \right) r^7 (r - i \omega)} + \frac{3 \left(2 + \frac{i q^2 \left(1 - \frac{1}{r^3} \right)^2}{9 (r - i \omega) \omega} - \frac{i \omega}{r} + \frac{1}{3} \left(1 - \frac{1}{r^3} \right) \left(-2 + \frac{q^2 + i r \omega}{r (r - i \omega)} \right) \right) hrr'[r]}{\left(1 - \frac{1}{r^3} \right) r} - \\
& (i (q^2 - 3 i r \omega) (6 i q^2 \omega hL[r] + 18 i r^2 \omega hL[r] - 24 i q^2 r^3 \omega hL[r] + 36 i r^3 \omega^3 hL[r] - 18 q^2 r^5 hrr[r] + \\
& \quad 18 q^2 r^8 hrr[r] - 9 r^2 \omega^2 hrr[r] + 36 r^5 \omega^2 hrr[r] - 27 r^8 \omega^2 hrr[r] + 3 i \omega hL'[r] + \\
& \quad 6 i q^2 r \omega hL'[r] + 6 i r^3 \omega hL'[r] - 6 i q^2 r^4 \omega hL'[r] + 9 i r^4 \omega^3 hL'[r] + q^2 hrr'[r] - \\
& \quad 3 q^2 r^6 hrr'[r] + 2 q^2 r^9 hrr'[r] - 3 r^3 \omega^2 hrr'[r] + 6 r^6 \omega^2 hrr'[r] - 3 r^9 \omega^2 hrr'[r])) / \\
& \left(3 \left(1 - \frac{1}{r^3} \right) r^7 (r - i \omega) \omega (q^2 + 2 q^2 r^3 - 3 i r \omega - 3 i q^2 r^2 \omega - 3 r^3 \omega^2) \right) + hrr''[r]
\end{aligned}$$

The substituted equation for the first order derivative of the equation for hrr[r]:

$$\left\{ \frac{-3 (q^2 + r (-4 r + 3 i \omega)) hrr[r] + (1 + 8 r^3 - 3 i r^2 \omega) hrr'[r] + r (-1 + r^3) hrr''[r]}{r (-1 + r^3)} \right\} [r]$$

The second order coefficient: {1}

Out[]= {0} == 0

The first order coefficient: $\left\{ -\frac{1 + 8 r^3 - 3 i r^2 \omega}{r - r^4} \right\}$

Out[]= True

The function expression: $\left\{ \frac{3 (q^2 + r (-4 r + 3 i \omega))}{r - r^4} \right\}$

Out[]= True