

BOLTZMANN EQUATION AND MOMENT EQUATIONS IN CURVILINEAR COORDINATES

E. M. Shakhov

Izv. AN SSSR. Mekhanika Zhidkosti i Gaza, Vol. 2, No. 2, pp. 155-160, 1967

ABSTRACT: Various forms of writing the Boltzmann equation in an arbitrary orthogonal curvilinear coordinate system are discussed. The derivation is presented of a general transport equation and moment equations containing moments of the distribution function no higher than the fourth. For a gas of Maxwellian molecules it is shown that the system of moment equations for flows which differ little from equilibrium flows transforms into the system of hydrodynamic equations. The resulting equations may be useful in solving problems on motions of a rarefied gas by the moment methods. The results are valid for both the Boltzmann equation and model kinetic equations.

1. All the variations which take place in a rarefied gas in the absence of external forces are described by the Boltzmann equation

$$\frac{\partial f}{\partial t} + \xi_x \frac{\partial f}{\partial x} + \xi_y \frac{\partial f}{\partial y} + \xi_z \frac{\partial f}{\partial z} = J. \quad (1.1)$$

Here x, y, z are Cartesian coordinates of the physical space; ξ_x, ξ_y, ξ_z are components of the molecular velocity vector ξ on the coordinate axes with unit vectors i, j, k ; f is the distribution function, depending on $t, x, y, z, \xi_x, \xi_y, \xi_z$; J is the collision integral, which is some functional of f , whose concrete form is not of interest to us.

We shall transform the Boltzmann equation to an arbitrary orthogonal coordinate system. Let q_1, q_2, q_3 be curvilinear coordinates of the point x, y, z of the physical space, so that

$$x = x(q_1, q_2, q_3), y = y(q_1, q_2, q_3), z = z(q_1, q_2, q_3), \quad (1.2)$$

and, conversely,

$$q_i = q_i(x, y, z) \quad (i = 1, 2, 3). \quad (1.3)$$

Converting in (1.1) from the Cartesian coordinates x, y, z to the curvilinear coordinates with unchanged ξ_x, ξ_y, ξ_z , we obtain

$$\frac{\partial f}{\partial t} + \frac{\xi_1}{H_1} \frac{\partial f}{\partial q_1} + \frac{\xi_2}{H_2} \frac{\partial f}{\partial q_2} + \frac{\xi_3}{H_3} \frac{\partial f}{\partial q_3} = J, \quad (1.4)$$

$$\xi_i = \frac{H_i \partial q_i}{\partial x} \xi_x + \frac{H_i \partial q_i}{\partial y} \xi_y + \frac{H_i \partial q_i}{\partial z} \xi_z,$$

$$H_i = \sqrt{(\partial x / \partial q_i)^2 + (\partial y / \partial q_i)^2 + (\partial z / \partial q_i)^2} \quad (i = 1, 2, 3). \quad (1.5)$$

Here H_i are the Lamé parameters.

The quantity $H_i \partial q_i$ is the length of the arc ∂s_i of the coordinate line q_i , and $H_i \partial q_i / \partial x$ is the cosine of the angle which the normal to the surface $q_i = \text{const}$, and consequently (in view of the system orthogonality) the line q_i as well, forms with the x -axis. Consequently, ξ_i is the projection of the velocity vector ξ on the tangent to the coordinate line q_i . It is obvious that for the same vector ξ the projections ξ_i at different points of the physical space are different. Denoting by e_1, e_2, e_3 the unit vectors of the curvilinear coordinate system, we write the expression for ξ_i in the form

$$\xi_i = \xi_x(e_i \cdot i) + \xi_y(e_i \cdot j) + \xi_z(e_i \cdot k) \quad (i = 1, 2, 3). \quad (1.6)$$

Thus, if the gas motion is considered in the orthogonal curvilinear coordinate system, and the molecular velocities are considered in the Cartesian frame, then for the distribution function $f(t, q_1, q_2, q_3, \xi_x, \xi_y, \xi_z)$ we have the Boltzmann equation in the form of (1.4), and ξ_i are expressed by Eq. (1.6).

To obtain the transport equations it is advisable to write the Boltzmann equation in divergence form.

To do this, we note that for the fixed vector

$$\xi = \xi_x i + \xi_y j + \xi_z k = \xi_1 e_1 + \xi_2 e_2 + \xi_3 e_3$$

we have the equality

$$\frac{1}{H_1 H_2 H_3} \left(\frac{\partial \xi_1 H_2 H_3}{\partial q_1} + \frac{\partial \xi_2 H_3 H_1}{\partial q_2} + \frac{\partial \xi_3 H_1 H_2}{\partial q_3} \right) = 0. \quad (1.7)$$

Equation (1.7) may be verified by direct differentiation, if we use the formulas for the derivatives $\partial \xi_i / \partial q_j$ which are presented below. However, it is sufficiently obvious physically, since the flux of a constant vector through any closed surface is equal to zero. Considering (1.7), we write the Boltzmann equation in the form

$$\frac{\partial f}{\partial t} + \frac{1}{H_1 H_2 H_3} \left(\frac{\partial f \xi_1 H_2 H_3}{\partial q_1} + \frac{\partial f \xi_2 H_3 H_1}{\partial q_2} + \frac{\partial f \xi_3 H_1 H_2}{\partial q_3} \right) = J. \quad (1.8)$$

This equation may also be derived directly, forming, just as in Cartesian coordinates, the balance of the number of particles of velocity $\xi = \xi_x i + \xi_y j + \xi_z k$ for the elemental volume $d\tau = H_1 H_2 H_3 dq_1 dq_2 dq_3$.

The conversion to curvilinear coordinates may be carried even further. Namely, not only is a point of the physical space specified by the coordinates q_1, q_2, q_3 , but the molecular velocity is also referred to the curvilinear coordinates, i. e., in place of the arguments $t, q_1, q_2, q_3, \xi_x, \xi_y, \xi_z$ in the distribution function we introduce $t, q_1, q_2, q_3, \xi_1, \xi_2, \xi_3$. Converting in (1.4) from ξ_x, ξ_y, ξ_z to ξ_1, ξ_2, ξ_3 according to (1.6), and noting that the collision integral is invariant with respect to axis rotation, we obtain

$$\frac{\partial f}{\partial t} + \xi_\alpha \frac{\partial f}{\partial s_\alpha} + \xi_\alpha \frac{\partial \xi_\beta}{\partial s_\alpha} \frac{\partial f}{\partial \xi_\beta} = J, \quad \frac{\partial}{\partial s_1} = \frac{1}{H_1} \frac{\partial}{\partial q_1}, \quad \text{etc.} \quad (1.9)$$

The derivatives of f with respect to q_i are taken for fixed ξ_1, ξ_2, ξ_3 , and those of ξ_β with respect to q_α are taken for fixed ξ_x, ξ_y, ξ_z . Here and everywhere hereafter the summation is understood over repeating Greek subscripts.

Thus, inertia terms appear in the Boltzmann equation. In order to give (1.9) its final form, we must substitute in place of $\partial \xi_\beta / \partial s_\alpha$ their expressions in terms of ξ_i, q_i (see Eqs. (2.4)); however, in view of the complexity this equation will not be written out.

2. To derive the general transport equation we use the Boltzmann equation in the divergence form of (1.8). We note that the transport equation may also be obtained from (1.9).

Let $\varphi(\xi_1, \xi_2, \xi_3)$ be an arbitrary function of ξ_1, ξ_2, ξ_3 . Multiplying (1.8) by φ and transforming the sum of the derivatives of f to the divergence form, we obtain

$$\frac{\partial f \varphi}{\partial t} + \frac{1}{H_1 H_2 H_3} \left(\frac{\partial f \varphi \xi_1 H_2 H_3}{\partial q_1} + \frac{\partial f \varphi \xi_2 H_3 H_1}{\partial q_2} + \frac{\partial f \varphi \xi_3 H_1 H_2}{\partial q_3} \right) - f \Phi = \varphi J, \quad (2.1)$$

$$\Phi = \frac{\xi_1}{H_1} \frac{\partial \varphi}{\partial q_1} + \frac{\xi_2}{H_2} \frac{\partial \varphi}{\partial q_2} + \frac{\xi_3}{H_3} \frac{\partial \varphi}{\partial q_3}. \quad (2.2)$$

The function φ depends on the coordinates only through the velocity components, so that

$$\frac{\partial \varphi}{\partial q_i} = \frac{\partial \varphi}{\partial \xi_\alpha} \frac{\partial \xi_\alpha}{\partial q_i} \quad (i = 1, 2, 3).$$

Consequently, Φ is represented in the form

$$\Phi = \frac{\partial \varphi}{\partial \xi_\alpha} \left(\xi_\beta \frac{\partial \xi_\alpha}{\partial s_\beta} \right). \quad (2.3)$$

The velocity $\xi = \xi_x i + \xi_y j + \xi_z k$ is an independent variable function of the distribution; therefore $\partial \xi / \partial q_i = 0$, and the derivatives $\partial \xi_i / \partial q_j$

are nonzero because of the curvilinearity of the coordinate system

$$\frac{\partial \xi_i}{\partial q_j} = \frac{\partial (\mathbf{e}_i \cdot \xi)}{\partial q_j} = \frac{\partial \mathbf{e}_i}{\partial q_j} \cdot \xi.$$

Using the formulas for the derivatives of unit vectors with respect to the coordinates q_j (see, for example, [1], page 206), we obtain

$$\begin{aligned} \frac{\partial \xi_1}{\partial q_1} &= -\frac{\xi_2}{H_1 H_2} \frac{\partial H_1}{\partial q_2} - \frac{\xi_3}{H_1 H_3} \frac{\partial H_1}{\partial q_3}, \quad \frac{\partial \xi_1}{\partial q_2} = \frac{\xi_2}{H_2 H_1} \frac{\partial H_2}{\partial q_1}, \\ \frac{\partial \xi_1}{\partial q_3} &= \frac{\xi_3}{H_3 H_1} \frac{\partial H_3}{\partial q_1}, \end{aligned} \quad (2.4)$$

and two groups of analogous expressions for the derivatives of ξ_2 and ξ_3 , which are obtained from (2.4) by cyclic permutation of subscripts.

Substituting into (2.3) the expressions for the derivatives of $\xi_{\alpha i}$ according to Eqs. (2.4), we represent Φ in the form

$$\Phi = \sum_{i,j=1}^3 \left(\frac{\xi_i^2}{H_i H_j} \frac{\partial H_i}{\partial q_j} - \frac{\xi_j \xi_i}{H_j H_i} \frac{\partial H_j}{\partial q_i} \right) \frac{\partial \varphi}{\partial \xi_j}. \quad (2.5)$$

We shall denote by angle brackets the averaged, macroscopic characteristics obtained from microscopic ones by multiplying by the distribution function f and integrating over the entire velocity space. Then, in accordance with (2.1), the general transport equation is written in the form

$$\begin{aligned} \frac{\partial \langle \varphi \rangle}{\partial t} + \frac{1}{H_1 H_2 H_3} \left(\frac{\partial \langle \varphi \xi_1 \rangle H_2 H_3}{\partial q_1} + \frac{\partial \langle \varphi \xi_2 \rangle H_3 H_1}{\partial q_2} + \frac{\partial \langle \varphi \xi_3 \rangle H_1 H_2}{\partial q_3} \right) - \\ - \langle \Phi \rangle = \langle J_{\varphi} \rangle, \quad \langle J_{\varphi} \rangle = \int \varphi J d\xi. \end{aligned} \quad (2.6)$$

3. We shall derive from the general transport equation (2.6) the moment equations, containing moments up to and including the fourth. Following [2], we introduce the notations

$$\begin{aligned} u_i = \langle \xi_i \rangle, \quad c_i = \xi_i - u_i, \quad P_{ij} = \langle c_i c_j \rangle, \quad p_{ij} = P_{ij} - \delta_{ij} p \\ S_{ijk} = \langle c_i c_j c_k \rangle, \quad S_i = S_{i\alpha\alpha}, \quad Q_{ijkl} = \langle c_i c_j c_k c_l \rangle \\ \langle J_{ij}^{(2)} \rangle = \int c_i c_j J d\xi, \quad \langle J_{ijk}^{(3)} \rangle = \int c_i c_j c_k J d\xi. \end{aligned}$$

In addition, we denote by $\langle \Phi_i^{(1)} \rangle$, $\langle \Phi_{ij}^{(2)} \rangle$, $\langle \Phi_{ijk}^{(3)} \rangle$ the averaged values of Φ for

$$\varphi = \xi_i, \quad \varphi = \xi_i \xi_j, \quad \varphi = \xi_i \xi_j \xi_k.$$

Setting $\varphi = 1$, $\varphi = \xi_i^2$ and $\varphi = \xi_1^2 + \xi_2^2 + \xi_3^2$ in the general transport equation, we obtain the mass, momentum, and energy conservation equations:

$$\frac{\partial \rho}{\partial t} + \frac{1}{H_1 H_2 H_3} \left(\frac{\partial \rho u_1 H_2 H_3}{\partial q_1} + \frac{\partial \rho u_2 H_3 H_1}{\partial q_2} + \frac{\partial \rho u_3 H_1 H_2}{\partial q_3} \right) = 0, \quad (3.1)$$

$$\begin{aligned} \frac{\partial \rho u_i}{\partial t} + \frac{1}{H_1 H_2 H_3} \left[\frac{\partial}{\partial q_1} H_2 H_3 (\rho u_1 u_i + P_{1i}) + \frac{\partial}{\partial q_2} H_3 H_1 (\rho u_2 u_i + P_{2i}) + \right. \\ \left. + \frac{\partial}{\partial q_3} H_1 H_2 (\rho u_3 u_i + P_{3i}) \right] - \langle \Phi_i^{(1)} \rangle = 0, \end{aligned} \quad (3.2)$$

$$\begin{aligned} \frac{\partial (3p + \rho u_{\alpha} u_{\alpha})}{\partial t} + \frac{1}{H_1 H_2 H_3} \left[\frac{\partial}{\partial q_1} H_2 H_3 (\rho u_1 u_{\alpha} u_{\alpha} + 3u_1 p + 2u_{\alpha} P_{1\alpha} + S_1) + \right. \\ \left. + \frac{\partial}{\partial q_2} H_3 H_1 (\rho u_2 u_{\alpha} u_{\alpha} + 3u_2 p + 2u_{\alpha} P_{2\alpha} + S_2) + \right. \\ \left. + \frac{\partial}{\partial q_3} H_1 H_2 (\rho u_3 u_{\alpha} u_{\alpha} + 3u_3 p + 2u_{\alpha} P_{3\alpha} + S_3) \right] = 0. \end{aligned} \quad (3.3)$$

The expressions for $\langle \Phi_i^{(1)} \rangle$ are easily obtained from (2.5):

$$\langle \Phi_i^{(1)} \rangle = -(\rho u_1 u_2 + P_{12}) \frac{1}{H_1 H_2} \frac{\partial H_1}{\partial q_2} - (\rho u_1 u_3 + P_{13}) \frac{1}{H_1 H_3} \frac{\partial H_1}{\partial q_3} +$$

$$+ (\rho u_2^2 + P_{22}) \frac{1}{H_2 H_1} \frac{\partial H_2}{\partial q_1} + (\rho u_3^2 + P_{33}) \frac{1}{H_3 H_1} \frac{\partial H_3}{\partial q_1}, \quad (3.4)$$

as are the analogous expressions for $\langle \Phi_2^{(1)} \rangle$ and $\langle \Phi_3^{(1)} \rangle$. Using the equation of continuity and the momentum equation, we write the energy in the form

$$\begin{aligned} \frac{\partial p}{\partial t} + \frac{1}{H_1 H_2 H_3} \left[\frac{\partial}{\partial q_1} H_2 H_3 \left(u_1 p + \frac{1}{3} S_1 \right) + \frac{\partial}{\partial q_2} H_3 H_1 \left(u_2 p + \frac{1}{3} S_2 \right) + \right. \\ \left. + \frac{\partial}{\partial q_3} H_1 H_2 \left(u_3 p + \frac{1}{3} S_3 \right) \right] + \frac{2}{3} P_{\alpha\beta} \frac{\partial u_{\alpha}}{\partial s_{\beta}} + \frac{2}{3} u_{\alpha} \langle \Phi_{\alpha}^{(1)} \rangle = 0. \end{aligned} \quad (3.5)$$

Now we set $\varphi = \xi_i \xi_j$ and $\varphi = \xi_i \xi_j \xi_k$ in (2.6). Performing the same transformations as in the derivation of the corresponding equations in Cartesian coordinates [2], we obtain

$$\begin{aligned} \frac{\partial P_{ij}}{\partial t} + \frac{1}{H_1 H_2 H_3} \left[\frac{\partial}{\partial q_1} H_2 H_3 (u_1 P_{ij} + S_{ij1}) + \frac{\partial}{\partial q_2} H_3 H_1 (u_2 P_{ij} + S_{ij2}) + \right. \\ \left. + \frac{\partial}{\partial q_3} H_1 H_2 (u_3 P_{ij} + S_{ij3}) \right] + P_{j\alpha} \frac{\partial u_i}{\partial s_{\alpha}} + P_{\alpha i} \frac{\partial u_j}{\partial s_{\alpha}} - \\ - \langle \Phi_{ij}^{(2)} \rangle + u_{\psi} \langle \Phi_{ij}^{(1)} \rangle + u_i \langle \Phi_j^{(1)} \rangle = I_{ij}^{(2)} \end{aligned} \quad (3.6)$$

$$\begin{aligned} \frac{\partial S_{ijk}}{\partial t} + \frac{1}{H_1 H_2 H_3} \left[\frac{\partial}{\partial q_1} H_2 H_3 (u_1 S_{ijk} + Q_{ijk1}) + \frac{\partial}{\partial q_2} H_3 H_1 (u_2 S_{ijk} + Q_{ijk2}) + \right. \\ \left. + \frac{\partial}{\partial q_3} H_1 H_2 (u_3 S_{ijk} + Q_{ijk3}) \right] + S_{i\alpha} \frac{\partial u_k}{\partial s_{\alpha}} + S_{i\alpha k} \frac{\partial u_j}{\partial s_{\alpha}} + S_{\alpha jk} \frac{\partial u_i}{\partial s_{\alpha}} - \\ - \frac{1}{\rho H_1 H_2 H_3} \left(P_{ij} \frac{\partial H_2 H_3 P_{1k}}{\partial q_1} + P_{jk} \frac{\partial H_2 H_3 P_{1i}}{\partial q_1} + P_{ki} \frac{\partial H_2 H_3 P_{1j}}{\partial q_1} + \right. \\ \left. + P_{ij} \frac{\partial H_3 H_1 P_{2k}}{\partial q_2} + P_{jk} \frac{\partial H_3 H_1 P_{2i}}{\partial q_2} + P_{ki} \frac{\partial H_3 H_1 P_{2j}}{\partial q_2} + \right. \\ \left. + P_{ij} \frac{\partial H_1 H_2 P_{3k}}{\partial q_3} + P_{jk} \frac{\partial H_1 H_2 P_{3i}}{\partial q_3} + P_{ki} \frac{\partial H_1 H_2 P_{3j}}{\partial q_3} \right) + \\ + \frac{1}{\rho} (P_{ij} \langle \Phi_k^{(1)} \rangle + P_{jk} \langle \Phi_i^{(1)} \rangle + P_{ki} \langle \Phi_j^{(1)} \rangle) - \langle \Phi_{ijk} \rangle + \\ + u_k \langle \Phi_{ij}^{(2)} \rangle + u_i \langle \Phi_{jk}^{(2)} \rangle + u_j \langle \Phi_{ki}^{(2)} \rangle - u_i u_j \langle \Phi_k^{(1)} \rangle - u_i u_k \langle \Phi_j^{(1)} \rangle - \\ - u_k u_i \langle \Phi_j^{(1)} \rangle = \langle J_{ijk}^{(3)} \rangle. \end{aligned} \quad (3.7)$$

The quantities $\langle \Phi_{ijk}^{(3)} \rangle$, $\langle \Phi_{ij}^{(2)} \rangle$, are calculated elementarily on the basis of (2.5); however, because of length they are not presented here.

4. Equations (3.6), (3.7) in the case of Maxwellian molecules make it possible to obtain expressions for the stresses and the thermal fluxes in the hydrodynamic approximation. Converting from P_{ij} to p_{ij} and excluding $\partial p / \partial t$ with the aid of the energy equation in the form (3.5), we write (3.6) in the form

$$\begin{aligned} \frac{\partial p_{ij}}{\partial t} + \frac{1}{H_1 H_2 H_3} \left[\frac{\partial}{\partial q_1} H_2 H_3 \left(u_1 p_{ij} + S_{ij1} - \frac{1}{3} \delta_{ij} S_1 \right) + \right. \\ \left. + \frac{\partial}{\partial q_2} H_3 H_1 \left(u_2 p_{ij} + S_{ij2} - \frac{1}{3} \delta_{ij} S_2 \right) + \right. \\ \left. + \frac{\partial}{\partial q_3} H_1 H_2 \left(u_3 p_{ij} + S_{ij3} - \frac{1}{3} \delta_{ij} S_3 \right) \right] + \\ + p_{j\alpha} \frac{\partial u_i}{\partial s_{\alpha}} + p_{\alpha i} \frac{\partial u_j}{\partial s_{\alpha}} - \frac{2}{3} \delta_{ij} p_{\alpha\beta} \frac{\partial u_{\alpha}}{\partial s_{\beta}} + \\ + p \left(\frac{\partial u_i}{\partial s_j} + \frac{\partial u_j}{\partial s_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_{\alpha}}{\partial s_{\alpha}} \right) - \\ - \langle \Phi_{ij}^{(2)} \rangle + u_j \langle \Phi_i^{(1)} \rangle + u_i \langle \Phi_j^{(1)} \rangle - \frac{2}{3} \delta_{ij} u_{\alpha} \langle \Phi_{\alpha}^{(1)} \rangle = \langle J_{ij}^{(2)} \rangle. \end{aligned} \quad (4.1)$$

For Maxwell molecules

$$\langle J_{ij}^{(2)} \rangle = -p \mu^{-1} p_{ij}$$

Passing in (4.1) to the limit as $\mu \rightarrow 0$ and combining terms in δ_{ij} , we find the expression for p_{ij} :

$$p_{ij} = -\mu \left[\frac{H_i}{H_j} \frac{\partial}{\partial q_j} \left(\frac{u_i}{H_i} \right) + \frac{H_j}{H_i} \frac{\partial}{\partial q_i} \left(\frac{u_j}{H_j} \right) \right] \quad (i \neq j), \quad (4.2)$$

$$p_{ii} = -\mu \left\{ 2 \left[\frac{u_\alpha \partial H_i}{H_i \partial s_\alpha} + \frac{\partial}{\partial q_i} \left(\frac{u_i}{H_i} \right) \right] - \frac{2}{3} \frac{1}{H_1 H_2 H_3} \left(\frac{\partial u_1 H_2 H_3}{\partial q_1} + \frac{\partial u_2 H_3 H_1}{\partial q_2} + \frac{\partial u_3 H_1 H_2}{\partial q_3} \right) \right\}. \quad (4.3)$$

Considering that the deformation rate tensor components are expressed by the equations [3]

$$2\varepsilon_{ij} = \frac{H_i}{H_j} \frac{\partial}{\partial q_j} \left(\frac{u_i}{H_i} \right) + \frac{H_j}{H_i} \frac{\partial}{\partial q_i} \left(\frac{u_j}{H_j} \right) \quad (i \neq j), \quad (4.4)$$

$$\varepsilon_{ii} = \frac{u_\alpha \partial H_i}{H_i \partial s_\alpha} + \frac{\partial}{\partial q_i} \left(\frac{u_i}{H_i} \right), \quad (4.5)$$

we obtain the linear connection between the stress tensor and the deformation rate tensor which is usual for Navier-Stokes theory:

$$p_{ij} = -2\mu\varepsilon_{ij}, \quad p_{ii} = -2\mu\varepsilon_{ii} + 2/3\mu\varepsilon_{\alpha\alpha} \quad (i \neq j) \quad (4.6)$$

We obtain the expression for the thermal flux components $S_i/2$ from (3.7). For Maxwellian molecules

$$\langle I_{ijk}^{(s)} \rangle = \frac{p}{6\mu} (S_i \delta_{jk} + S_j \delta_{ik} + S_k \delta_{ij} - 9S_{ijk}).$$

We set $j = k = \alpha$ in (2.8), sum with respect to α , and pass to the limit as $\mu \rightarrow 0$. Noting that in the limit

$$Q_{ijk\ell} = pRT (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{i\ell}\delta_{jk})$$

and that all the terms depending on Φ cancel one another, we arrive at the usual expression of the Fourier law for a monatomic gas:

$$^{1/2}S_i = -\lambda \frac{\partial T}{H_i \partial q_i} \quad (\lambda = ^{15}/_4 R\mu). \quad (4.7)$$

5. The transport equations and the moment equations presented above, written in universal form, may be useful for solving problems by the moment method, i. e., by the method of closing the system of moment equations by suitable approximation of the distribution function. In particular, we could write the 13-moment Grad equation, specifying, just as in [2], the form of the distribution function. However, this approach is not justified because of the asymptotic nature of the series convergence.

In conclusion we note that the form of the collisional term was not used in the arguments presented above, other than §4, so that the results are valid for both the Boltzmann equation and its kinetic models.

The author wishes to thank A. A. Nikol'skii for discussions and helpful comments.

REFERENCES

1. N. E. Kochin, Vector Calculus and Fundamentals of Tensor Calculus [in Russian], Izd-vo Nauka, 1965.
2. H. Grad, "On the kinetic theory of rarefied gas," Commun. on pure and appl. mathem., vol. 2, no. 4, 1949.
3. N. A. Stezkin, Dynamics of a Viscous Incompressible Fluid [in Russian], Gostekhizdat, 1955.

19 October 1966

Moscow