Comparison of Loading methods

This document compares the theoretical parameters to expect when loading a 6”x6”x0.06” sample of aluminum using various methods. A summary is at the end of the document.

List of Variables:

σ = Stress

L = Length of sample

w = Width of sample

h = Thickness of sample

f = Frequency of test run (up to 100 Hz)

F = Max Load applied

P = Power

U = Energy

δ = Displacement

E = Modulus of Elasticity ($10E^6 psi$)

I = Moment of Inertia, $\frac{wh^3}{12}$

Cantilever Beam:

$$\sigma = \frac{My}{I} = \frac{FL\,h/2}{I}$$

$$\delta_{max} = \frac{FL^3}{3EI} \qquad I = \frac{wh^3}{12}$$

$$\Rightarrow \sigma = \frac{FL\frac{h}{2}}{I} \Rightarrow F = \frac{2\sigma I}{Lh} = \frac{2\sigma \frac{h^3 w}{12}}{Lh} = \frac{h^2 w \sigma}{6L}$$

For extreme values
$h = 0.06"$, $w = 6"$, $\sigma = 40{,}000$ psi
$L = 6"$

Max Force $\Rightarrow$ $F_{max} = 24$ lb

Displacement

$$\delta_{max} = \frac{FL^3}{3EI} \Rightarrow \frac{24\text{ lb}\cdot 6"^3}{3\cdot 10E^6\text{ psi}\,\frac{(6\cdot .06^3)}{12}} = 0.46"$$

← Max Displacement

using $E = 10E^6$ psi

Power

using $F = \frac{2\sigma I}{Lh}$

$$U = \int_0^L \frac{M^2}{2EI}dx = \int \frac{F^2x^2}{2EI}dx = \frac{F^2x^3}{6EI}\Big|_0^L = \frac{F^2L^3}{6EI} = \frac{12F^2L^3}{6Ewh^3} = \frac{2\left(\frac{2\sigma I}{Lh}\right)^2 L^3}{Ewh^3} = \frac{8\sigma^2 I^2 L}{Ewh^5}$$

$$U = \frac{8\sigma^2 w^2 h^6 L}{144\,Ewh^5} = \frac{\sigma^2 whL}{18E}$$

Max Energy stored

$P = \frac{U}{t}$ t = Time to apply load $\therefore \frac{1}{4f}$

Estimated peak power

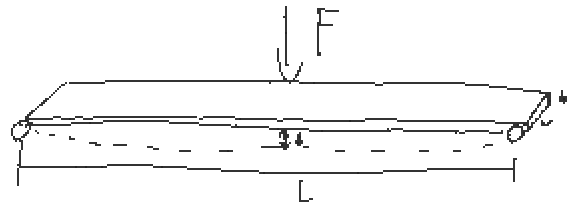
$$\Rightarrow P = \frac{2\sigma^2 whL}{9E}$$

Same as 3 Point!

3 Point Bend

$\delta = \frac{FL^3}{48EI}$ $M_{max} = \frac{FL}{4}$

$\sigma_{max} = \frac{FL\frac{h}{2}}{4I} = \frac{FLh}{8I}$



Force

$$F = \frac{8\sigma I}{Lh} = \frac{8}{12}\frac{\sigma w h^3}{Lh} = \frac{2}{3}\frac{\sigma w h^2}{L}$$

for values used above $\frac{2}{3}\frac{40E^3 psi \cdot 6'' \cdot 0.06''^2}{6''}$

$$\boxed{F_{max} = 96\ lb}$$

Displacement

$$\delta_{max} = \frac{FL^3}{48EI} \Rightarrow \frac{96\ lb \cdot 6''^3}{48 \cdot 10E^6 psi \left(\frac{6'' \cdot 0.06''^3}{12}\right)} = \boxed{0.6\bar{1}''}$$ Max displacement

Power $M = \frac{Fx}{2}$ ← M is Even function

$$U = \int_0^L \frac{M^2}{2EI}dx = 2\int_0^{L/2} \frac{F^2x^2}{8EI}dx = \frac{2}{8}\frac{F^2x^3}{3EI}\Bigg]_0^{L/2} = \frac{F^2L^3}{96EI}$$

$$\Rightarrow F = \frac{8\sigma I}{Lh} \Rightarrow U = \frac{64\sigma^2 I^2}{L^2h^2}\frac{L^3}{96EI}$$

$$= \frac{2}{3}\frac{\sigma^2 IL}{Eh^2} = \frac{\sigma^2 whL}{18E} =$$

(See above)

$$P = 4Uf = \boxed{\frac{2}{9}\frac{\sigma^2 whL}{E} = P_{max}}$$

Same as cantilever!

With $\sigma = 40E^3 psi$
$h = 0.06''$
$w = 6''$
$L = 6''$
$E = 10E^6 psi$
$f = 100\ Hz$

$$\boxed{P = 827.7\ W}$$

with $\sigma = 40E^3 psi$
$h = 0.06''$
$w = 1''$
$L = 6''$
$E = 10E^6 psi$
$f = 10\ Hz$

$$\boxed{P = 14.5W}$$

We won't do 100Hz with 40,000 psi!

Summary:

	Cantilever Beam	3 Point Bend
Force (lbf)	24	96
Max displacement (in)	0.44”	0.11”
Power using extreme values from spec, (w)	872.7	872.7
Expected Power (w)	14.5	14.5

Expected power was found with a 1” wide sample at 10hz. We do not need to hit “Power using extreme values”, it is simply the largest that could result from the extreme values in the spec. Typical values will be closer to “Expected Power”. Expected power was calculated using a 1” wide sample at 10hz, but smaller stresses would also lower the power needed.

The power requirements for both methods are the same. By comparing the max force and displacement, a Cantilever beam seems like the preferred choice. Smaller forces mean less stress on the fixture. Also, larger displacements make it easier to have a fine control over the load applied.

From these measurements, loading our sample as a cantilever beam seems like the better choice.

However, Dr. Bob Stephens recommended we use 3 point bending to apply a load.

An axial loading scenario was not included in this comparison because the force required to provide the necessary stresses is far too high (upwards of 600lb for a sample with a 1”x0.06” cross section), and buckling can occur at below 100lb.