



University of Idaho

College of Engineering



MASSPOINT DESIGN REVIEW

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Sponsored by Schweitzer Engineering Laboratories

OBJECTIVE OF PROJECT



- Need for a device to measure 3-axis Center of Gravity (CG).
- CG location is an important point for failure analysis.
- Current methods of determining CG are inadequate.
- Simplifies workflow for SEL employees.



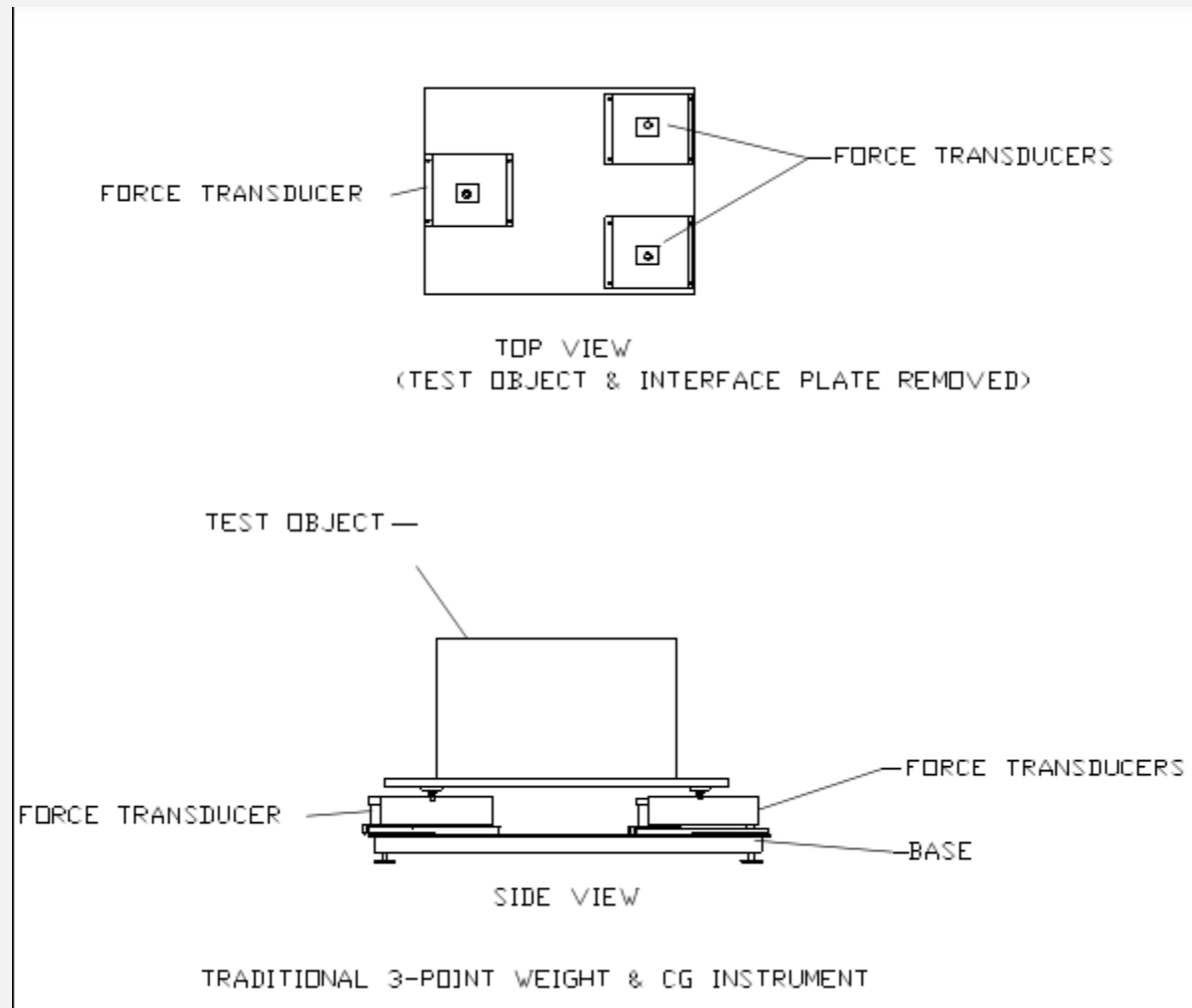
PRODUCT REQUIREMENTS



Create a device which:

- Calculates the 3-axis CG for complex objects.
- Measure objects of length $5'' < L < 20''$
- Automated operation.
- Axis coordinates within 3 seconds
- Accuracy of 1% or 0.1 inches.
- Hold a static load of 50 pounds.
- Operate 1 year without maintenance.
- Uses standard 120VAC.
- \$5000

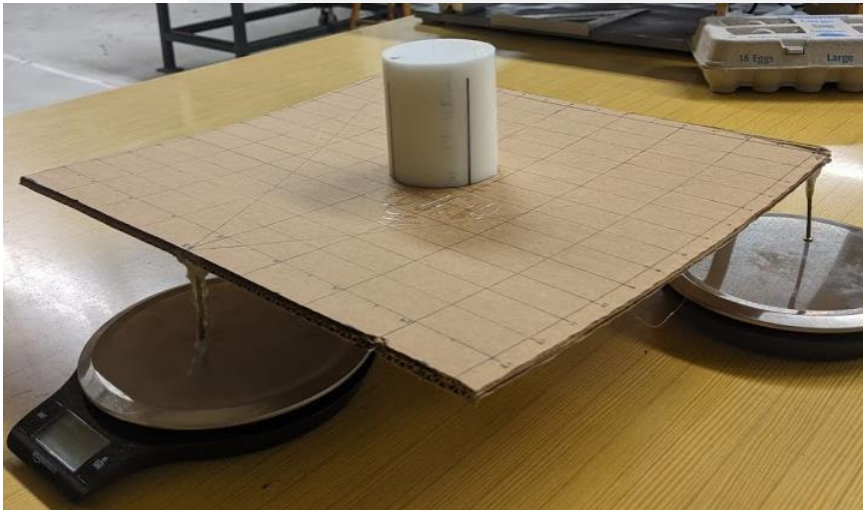
MULTIPLE POINT WEIGHING



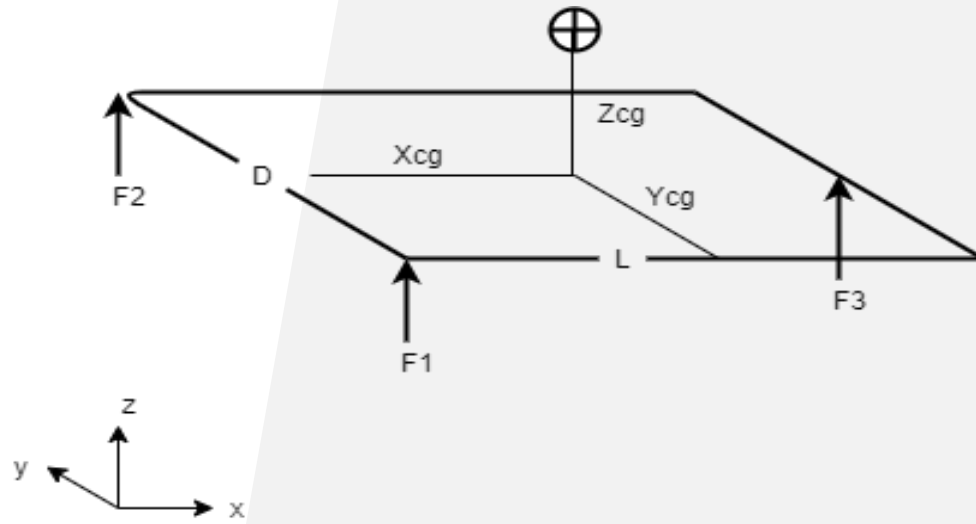
DATA ACQUISITION



- Three kitchen Scales.
- Cardboard tabletop surface.
- Nails.

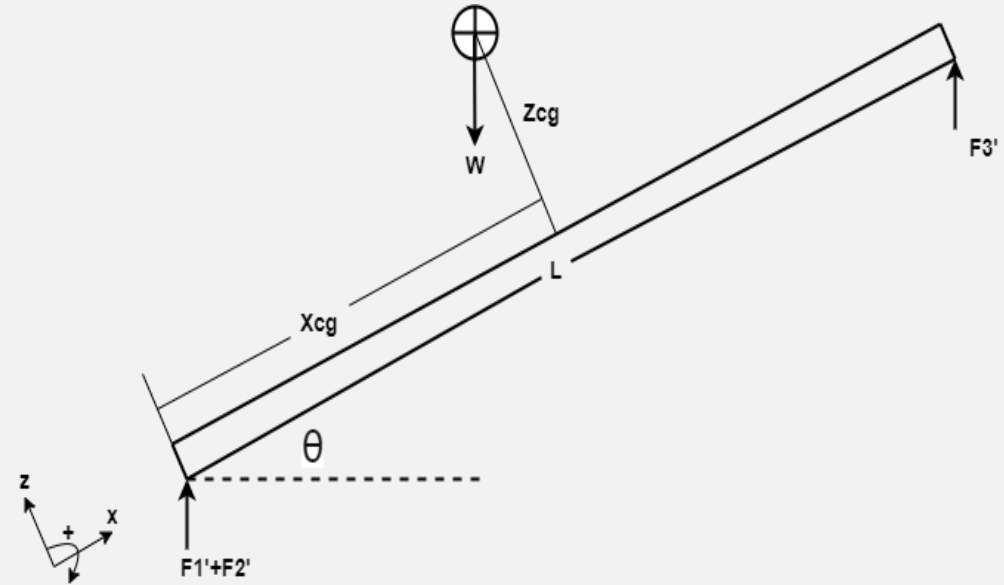


STATICS METHOD



$$x = \frac{LF_3}{w}$$

$$y = \frac{D(F_3 + 2F_2)}{2w}$$

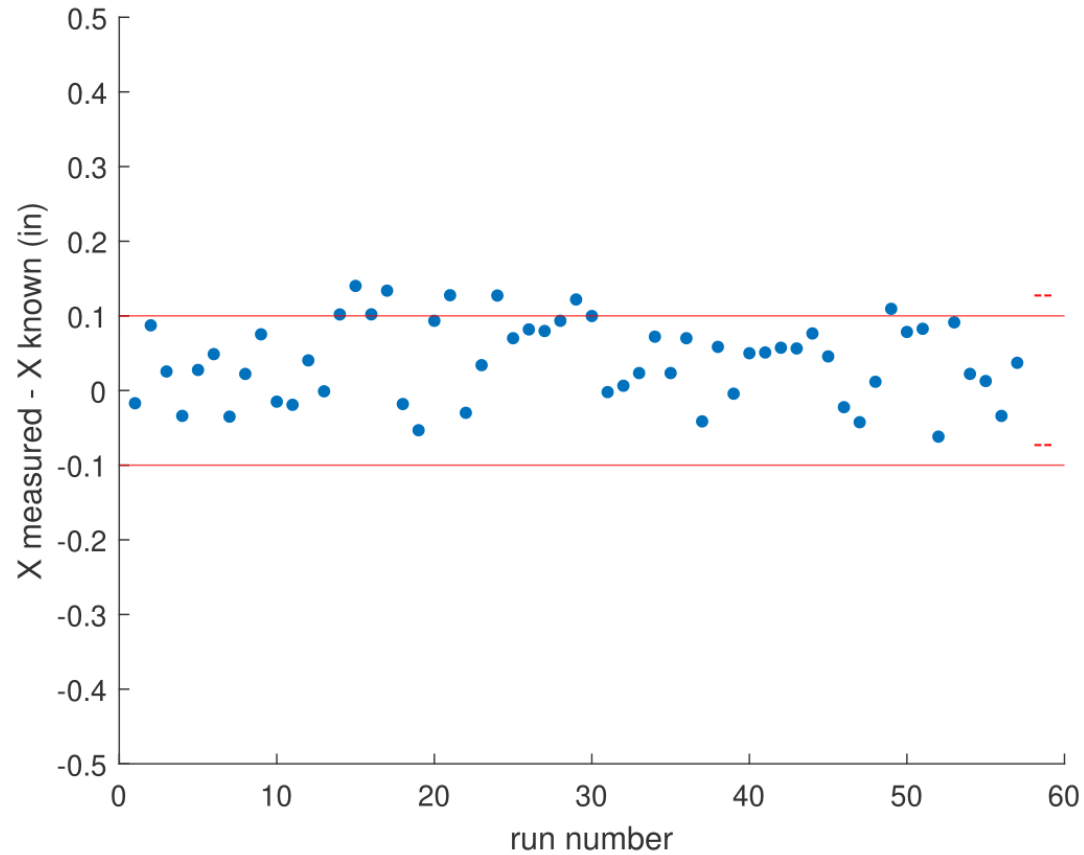


$$z = \frac{\cot(\theta)(w'x - F_3'L)}{w} - t$$

X AND Y PLOTS



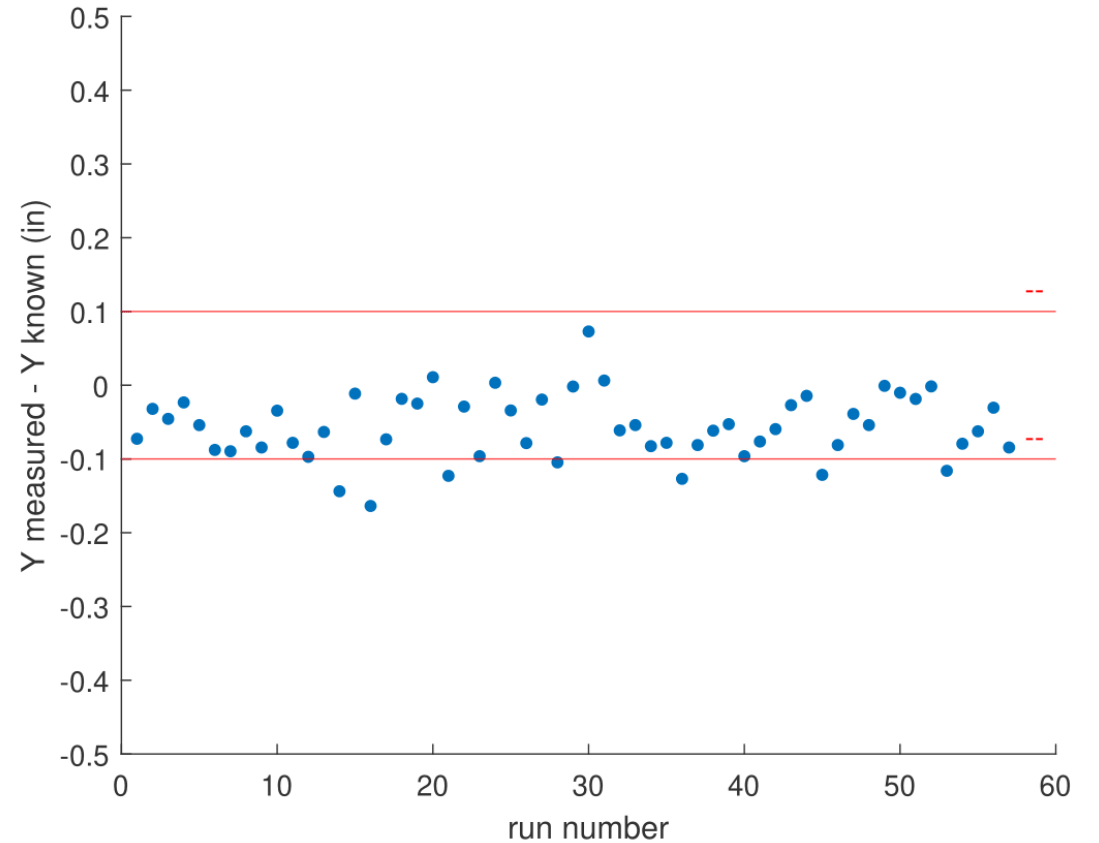
Error X Calculation



Mean Offset = 0.041in

$\sigma = \pm 0.054$ in

Error Y Calculation



Mean Offset = -0.057in

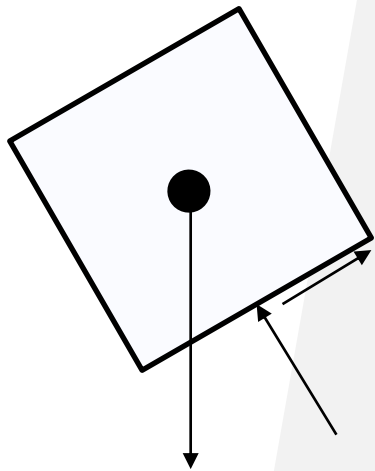
$\sigma = \pm 0.053$ in

STATICS METHOD: ERROR

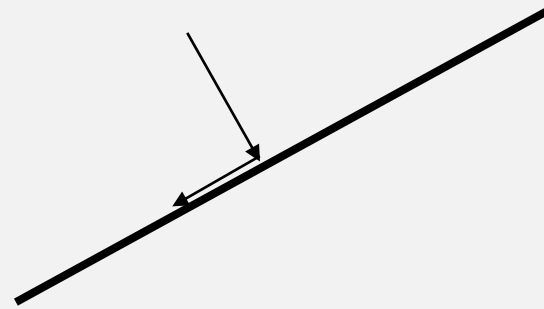


Why it doesn't work

- Certain FBD assumptions can't be made.
- 2 separate FBD are required.

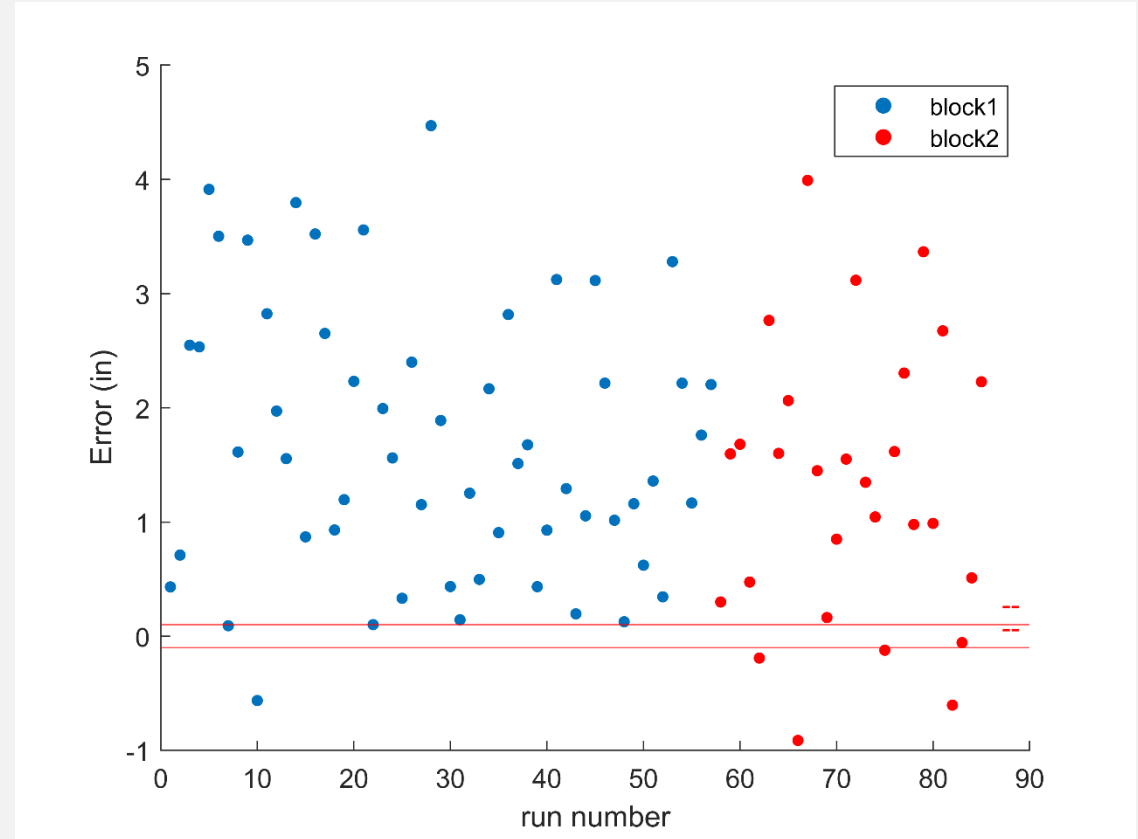


FBD: Object on table



FBD: Table

Difference Between Calculated and Known

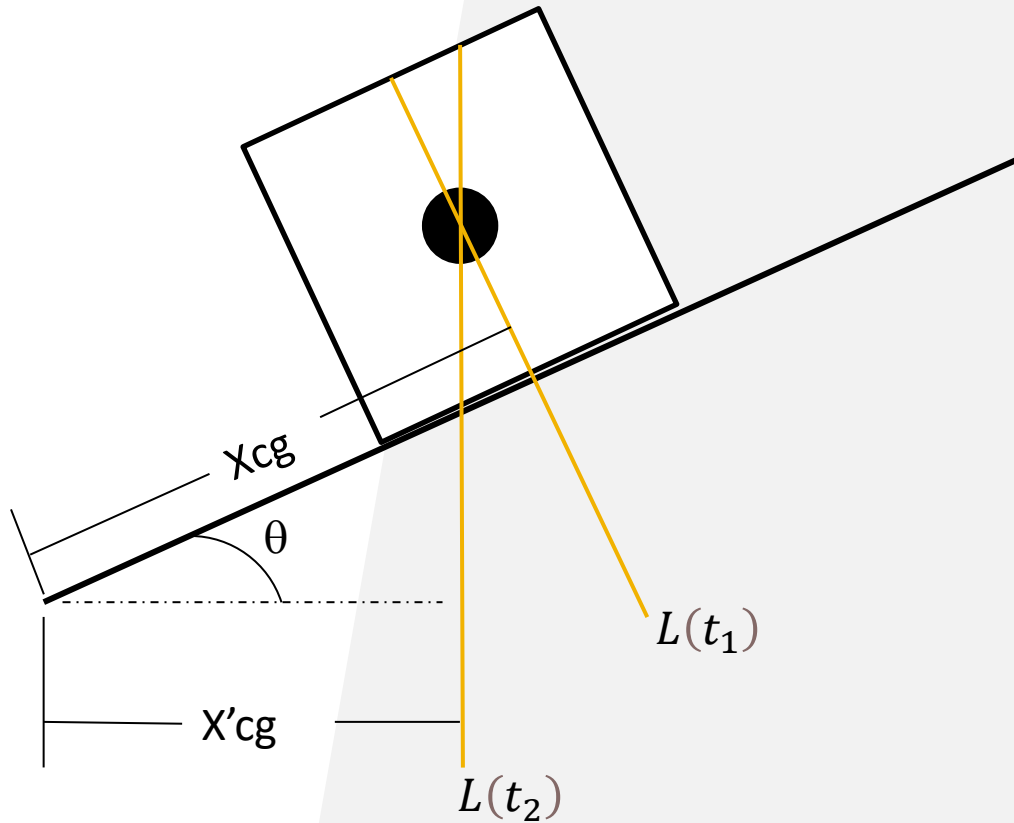


Mean Offset = 1.68 in $\sigma_1 : \pm 1.18$ in
Mean Offset = 1.31 in $\sigma_2 : \pm 1.22$ in

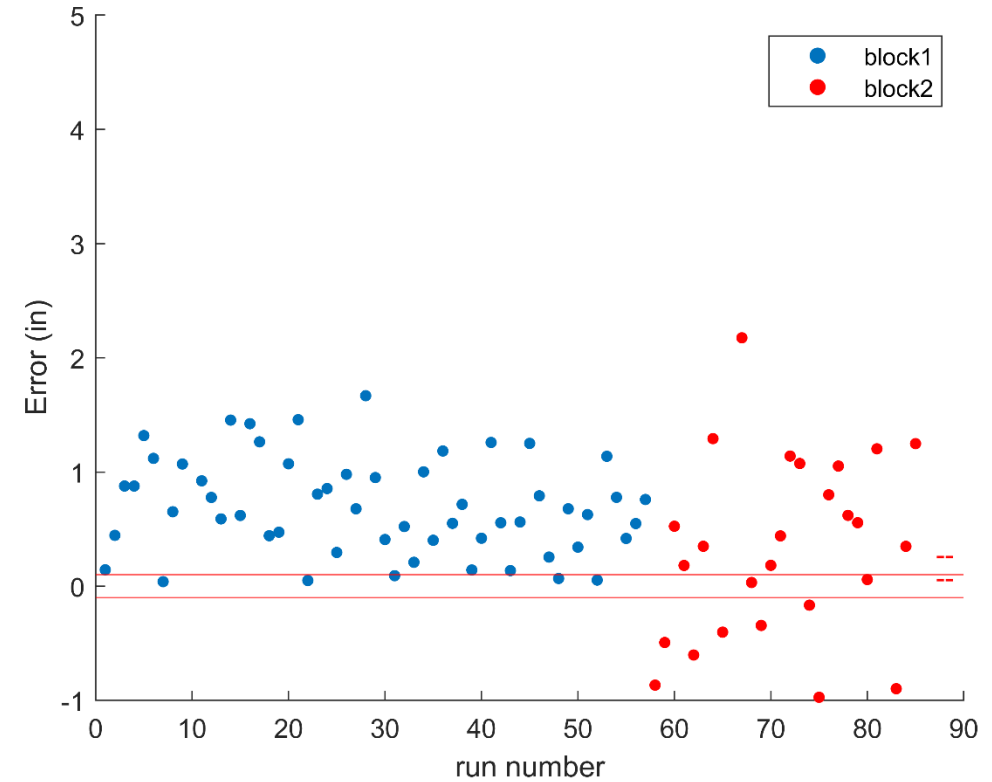
VECTOR LINE METHOD: ERROR



Difference Between Calculated and Known



Zcg: $L(t_1) = L(t_2)$

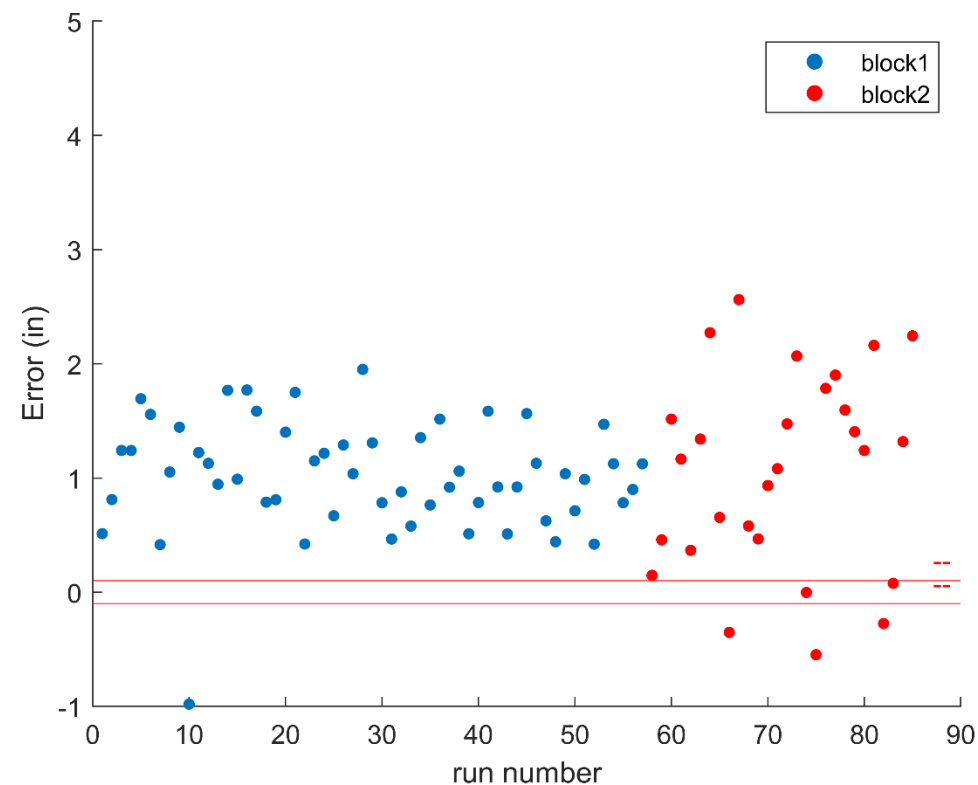
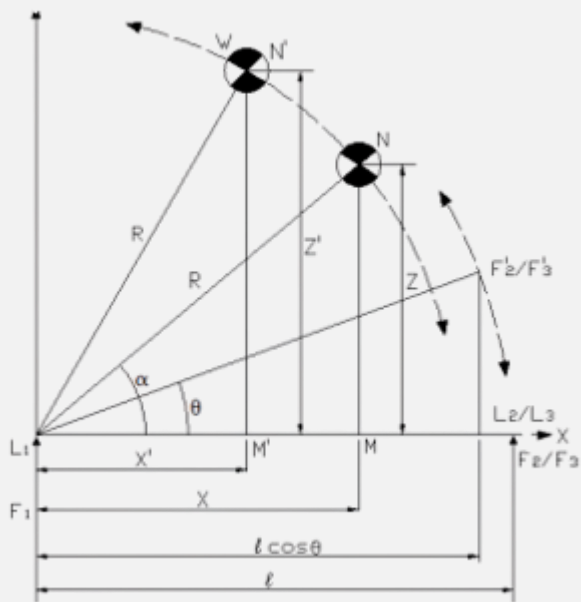


Mean Offset = 0.66 in $\sigma_1 : \pm 0.49$ in
Mean Offset = 0.21 in $\sigma_2 : \pm 0.87$ in

KUMAR, ET AL. METHOD

Difference Between Calculated and Known

- $X = \frac{(F_2 + F_3) * L}{W}$
- $Y = \frac{(F_3 + F_2) * D}{2W}$
- $X' = \frac{L * \cos(\theta) * (F_2' + F_3')}{W}$
- $\alpha = \tan^{-1}(\cot(\theta) - (\frac{X'}{X \sin(\theta)}))$
- $Z = R \sin(\alpha)$

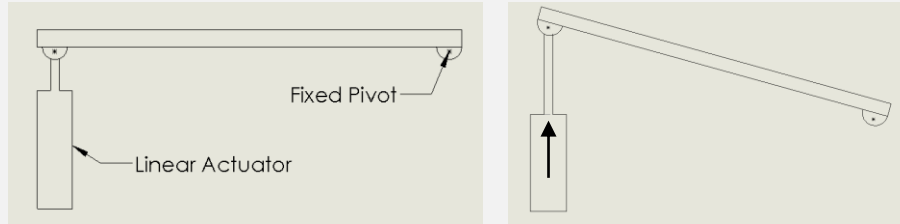


Mean Offset = 1.01 in $\sigma_1 : \pm 0.48$ in
 Mean Offset = 1.05 in $\sigma_2 : \pm 0.87$ in

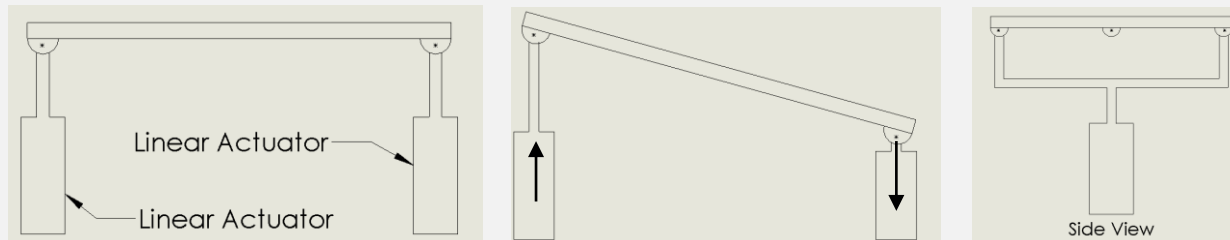
TABLE TILTING MECHANISM



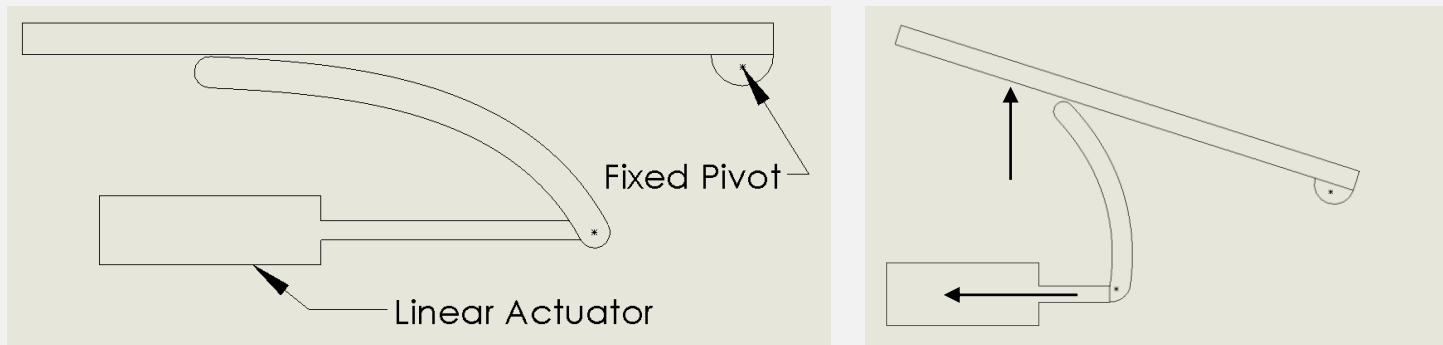
- Single Pushrod



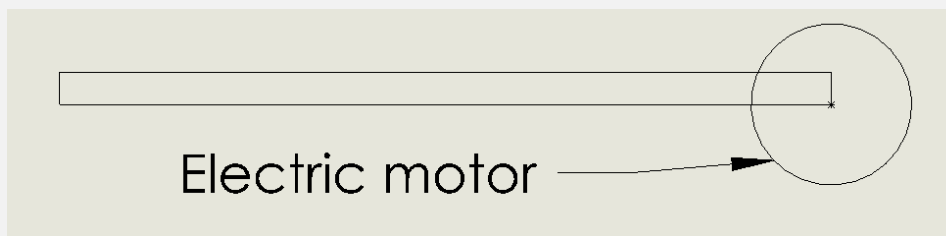
- Double Pushrod



- Cantilever



- Direct Torque



RECOMMENDATIONS



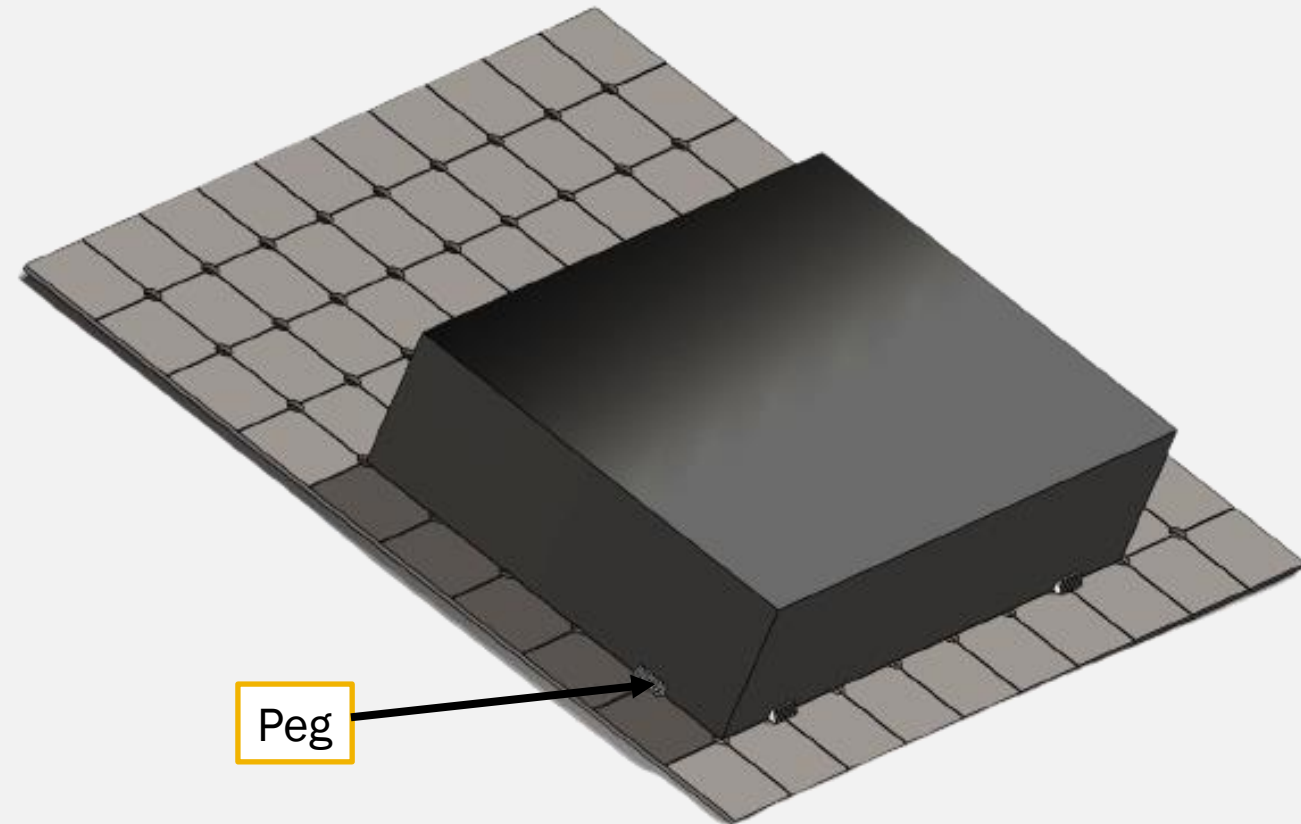
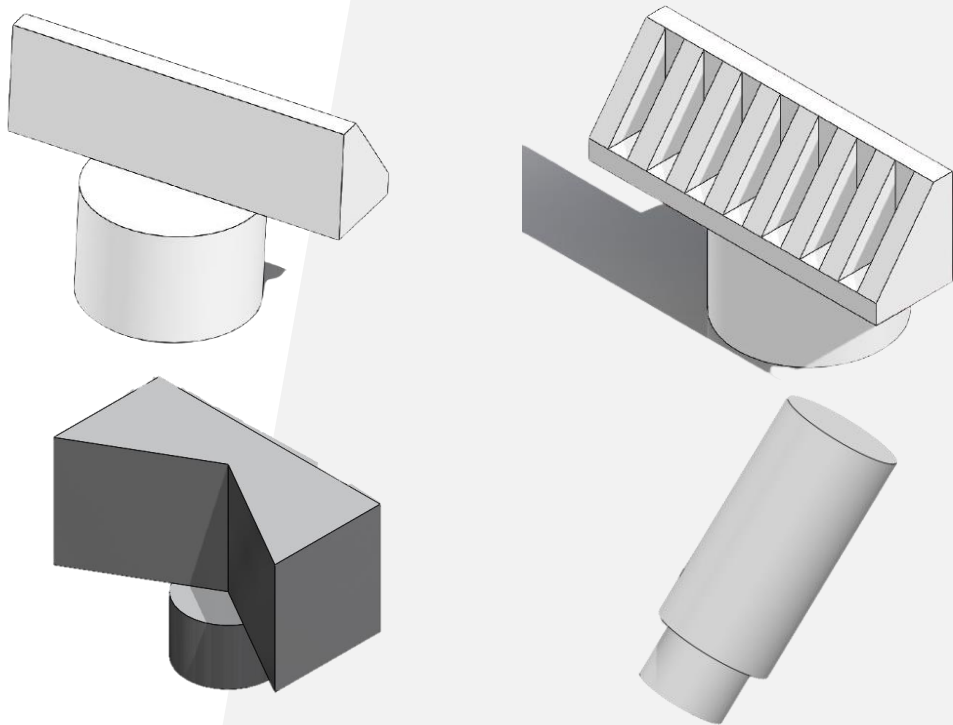
- Table Tilt Mechanism: Single Push Rod design to be carried out

	Multiplier	Pushrod	Double pushrod	Cantilever	Direct Torque	Pulley	Drawbridge	Ratchet Headrest
Efficiency/5	10	2	3	4	5	2	2	3
Cost/5	20	5	3	3	4	3	3	5
Size/5	20	2	2	5	4	3	4	5
Complexity/5	25	4	2	2	3	3	2	3
Reliability/5	25	5	4	3	3	5	4	2
Totals	100	77	56	65	72	68	62	71

TABLE-TOP DESIGN



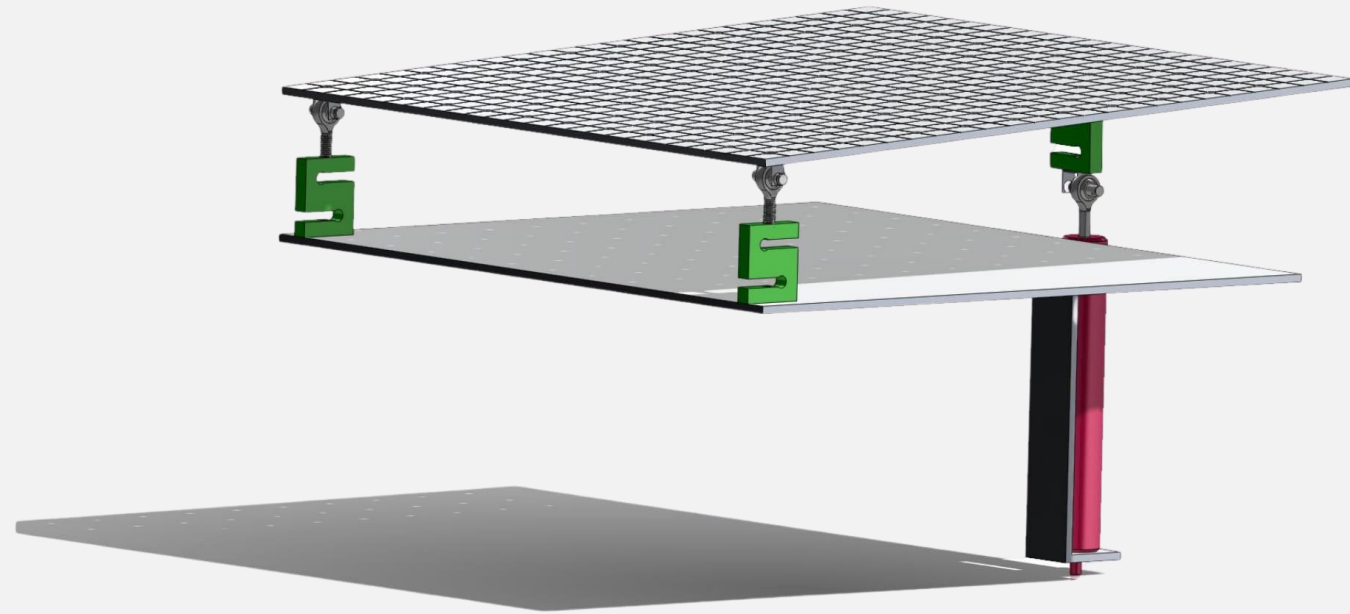
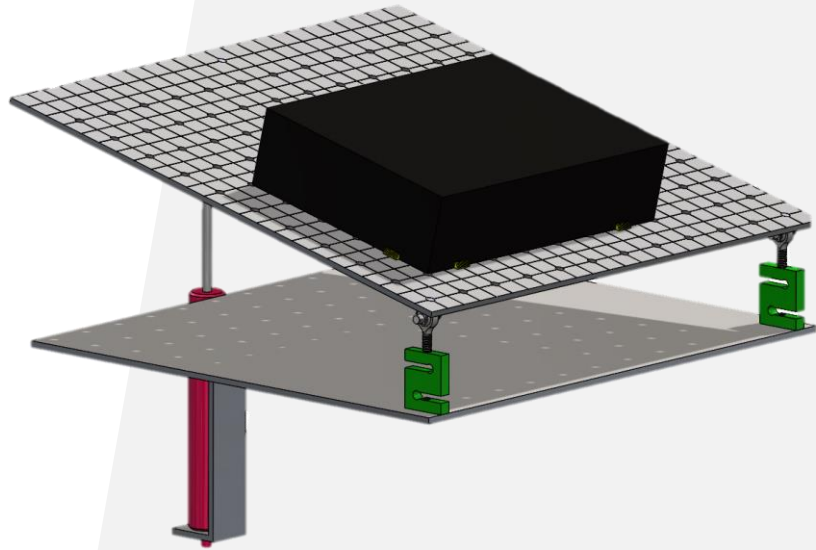
- Pegboard design which can hold multiple shape objects
- Lines engraved on surface to locate the test object
- Possibility to 3D print different peg shapes



SOLIDWORKS MOCK-UP



- Combine various aspects of our design
- Test different geometries



BUDGET



Spent so far

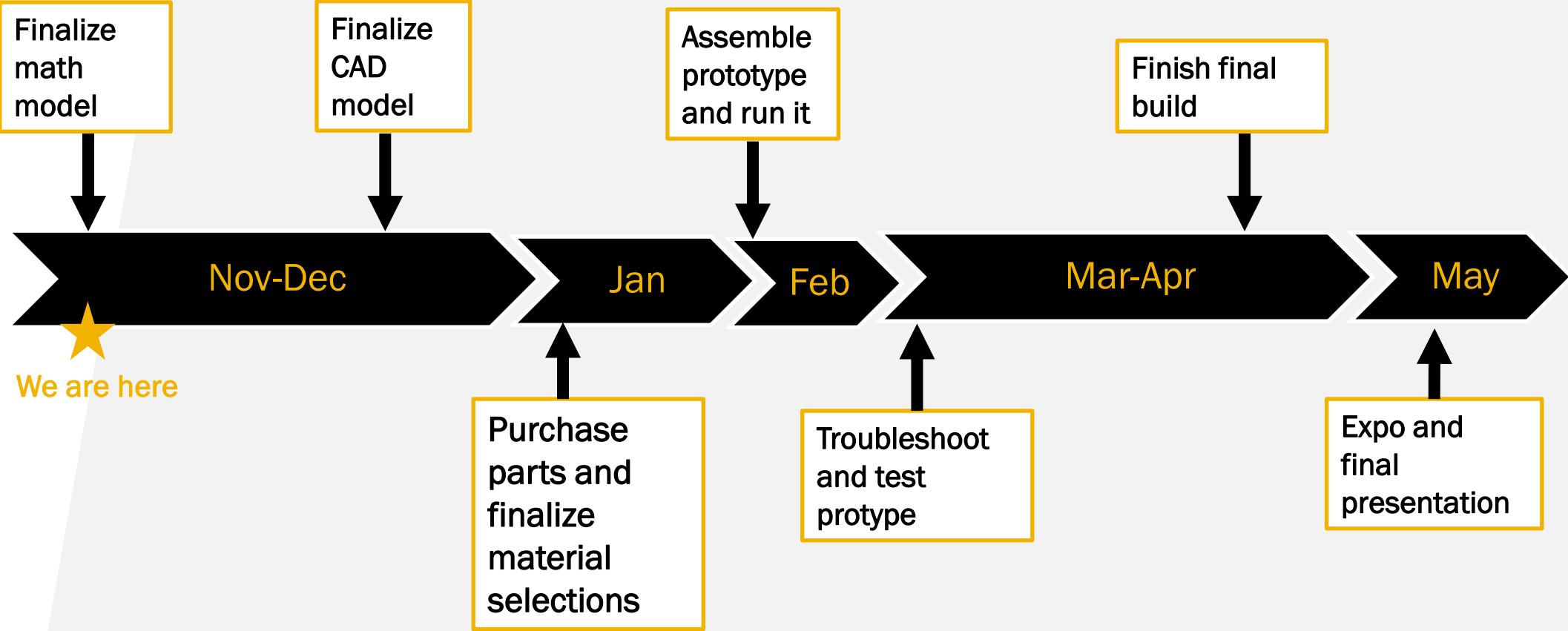
Item	Cost
Kitchen Scales	\$32.97
Spring Scales	\$17.82
Total	\$50.79

Going Forward

Item	Cost
Force sensors	\$1500
Aluminum	\$300
Fasteners	\$75
Electronics	\$100
Prototyping	\$100
Finishing	\$250
Total	\$2325

Remaining Balance: \$2449.21

SCHEDULE



QUESTIONS





Phase 1 Planning				08/Sep/20	13/Oct/20	36		
1	Team Contract	TEAM	-	08/Sep/20	10/Sep/20	3	09/Sep/20	-
2	Past Project Review	TEAM	-	09/Sep/20	15/Sep/20	7	15/Sep/20	-
3	Budget Draft	KS	MJ	11/Sep/20	17/Sep/20	7	16/Sep/20	16/Sep/20
4	Client Visit & Interview	JC	-	11/Sep/20	22/Sep/20	12	16/Sep/20	-
5	Product Requirements	PB	CE	16/Sep/20	24/Sep/20	9	16/Sep/20	16/Sep/20
6	Project Schedule	JC	MJ	22/Sep/20	06/Oct/20	15	06/Oct/20	06/Oct/20
7	Logbook	IND	-	07/Oct/20	08/Oct/20	2	08/Oct/20	-
8	Portfolio	-	-	06/Oct/20	08/Oct/20	3	08/Oct/20	-
9	Measurement Device options	KS	CE	17/Sep/20	22/Sep/20	6	22/Sep/20	22/Sep/20
10	CG measurement Research	IND	-	17/Sep/20	22/Sep/20	6	22/Sep/20	-
11	Design Suite table	PB	CE	22/Sep/20	24/Sep/20	3	22/Sep/20	22/Sep/20
12	Get old team prototype	CE	JC	24/Sep/20	29/Sep/20	6	28/Sep/20	29/Sep/20
13	Measurement of CG (Tension)	KS	CE JC	23/Sep/20	05/Oct/20	13	10/Sep/20	11/Sep/20
14	Measurement of CG (Compression)	MJ	PB JC	23/Sep/20	05/Oct/20	13	07/Oct/20	07/Oct/20
15	Recalculate Z cg equation	MJ	JC	06/Oct/20	08/Oct/20	3	08/Oct/20	09/Oct/20
16	Matlab code plotting error	JC	-	06/Oct/20	08/Oct/20	3	07/Oct/20	-
17	Small cube measurements	PB	MJ	06/Oct/20	08/Oct/20	3	08/Oct/20	08/Oct/20
18	Prototype brainstorm/review	TEAM	-	06/Oct/20	13/Oct/20	8	20/Oct/20	-
19	Prototype Selection	TEAM	-	08/Oct/20	13/Oct/20	6	20/Oct/20	-
20	Snapshot Presentation	CE	KS	07/Oct/20	09/Oct/20	3	12/Oct/20	12/Oct/20
21	Presentation Practice	CE	KS	09/Oct/20	12/Oct/20	4	12/Oct/20	12/Oct/20
22	Snapshot 1	TEAM	-	13/Oct/20	13/Oct/20	1	13/Oct/20	-
Phase 2 Concept Development				14/Oct/20	11/Dec/20	59		
23	Design Validation Plan	TEAM	-	14/Oct/20	22/Oct/20	9	22/Oct/02	-
24	Uncertainty analysis	MJ	JC	14/Oct/20	21/Oct/20	8	20/Oct/20	20/Oct/20
25	Method of holding parts to table	KS	CE	20/Oct/20	27/Oct/20	8	27/Oct/20	27/Oct/20
26	Lifting Mechanism Concept	PB	MJ	20/Oct/20	27/Oct/20	8	27/Oct/20	27/Oct/20
27	Finalize math model	JC	CE	14/Oct/20	29/Oct/20	16	29/Oct/20	29/Oct/20
28	Value Proposition	KS	CE	22/Oct/20	05/Nov/20	15	05/Nov/20	08/Nov/20
29	Lifting Mechanism Desision matrix	MJ	PB	27/Oct/20	29/Oct/20	3	29/Oct/20	29/Oct/20
30	Pegboard CAD model	CE	KS	27/Oct/20	29/Oct/20	3	29/Oct/20	29/Oct/20
31	Wikipage Draft	PB	KS	27/Oct/20	05/Nov/20	10	02/Nov/20	03/Nov/20
32	Assemble previous prototype	CE	JC	03/Nov/20	10/Nov/20	8	10/Nov/20	10/Nov/20
33	Verify Math model	JC	CE	05/Nov/20	12/Nov/20	8	11/Nov/20	11/Nov/20
34	Lifting Mechanism Design	MJ	PB KS	03/Nov/20	12/Nov/20	10	09/Nov/20	09/Nov/20
35	Design Review Presentation	TEAM	-	09/Nov/20	11/Nov/20	3		
36	Logbook	IND	-	11/Nov/20	12/Nov/20	2		
37	Portfolio	TEAM	-	10/Nov/20	12/Nov/20	3		
	Design solidworks			20/Oct/20	03/Nov/20	15		
	Build new design			27/Oct/20	20/Nov/20	25		
	Update control code			18/Oct/20	03/Nov/20	17		
	Arduino and sensors circuit			25/Oct/20	10/Nov/20	17		
	Snapshot Presentation			30/Nov/20	03/Dec/20	4		
	Snapshot 2			04/Dec/20	04/Dec/20	1		
	Logbook & Portfolio/Wiki			09/Dec/20	11/Dec/20	3		

Phase 3 System Level Design				05/Dec/20	05/Feb/21	63		
	Design Expo Registration			27/Jan/21	28/Jan/21	2		
	Uncertainty analysis load cells			13/Jan/21	19/Jan/21	7		
	Final sensor selection			19/Jan/21	20/Jan/21	2		
	Order load cells (8 wk lead)			21/Jan/21	21/Jan/21	1		
	Uncertainty analysis angle sensor			19/Jan/21	05/Feb/21	18		
	Automate code			13/Jan/21	05/Feb/21	24		
						-		
Phase 4 Detail Design				16/Jan/21	06/Mar/21	50		
	Engineering Release Review			19/Feb/21	21/Feb/21	3		
	Logbook	IND		20/Feb/21	21/Feb/21	2		
	Team Member Citizenship	IND		25/Feb/21	26/Feb/21	2		
	Load cell connection design			16/Jan/21	05/Feb/21	21		
	Material Selection (1)			16/Jan/21	25/Jan/21	10		
	Material Selection (2)			16/Jan/21	25/Jan/21	10		
	Material Selection (3)			25/Jan/21	05/Feb/21	12		
	Bearing selection (1)			16/Jan/21	21/Jan/21	6		
	Fasterner Selection (1)			17/Jan/21	23/Jan/21	7		
	Motor/Actuator selection			13/Jan/21	20/Jan/21	8		
	Order Materials (~2 week lead)			25/Jan/21	25/Jan/21	1		
	Assemble Prototype			11/Feb/21	01/Mar/21	19		
	Machine Part (1)			08/Feb/21	15/Feb/21	8		
	Machine Part (2)			08/Feb/21	15/Feb/21	8		
	Machine Part (3)			15/Feb/21	22/Feb/21	8		
	Install sensors			15/Feb/21	20/Feb/21	6		
	Snapshot Presentation			01/Mar/21	05/Mar/21	5		
	Snapshot 3			06/Mar/21	06/Mar/21	1		
Phase 5 Testing/Refinement				07/Mar/21	08/May/21	63		
	Portfolio & Wikipage			08/Mar/21	10/Mar/21	3		
	Logbook	IND		16/Apr/21	17/Apr/21	2		
	Fix bugs from snapshot			07/Mar/21	14/Mar/21	8		
	Reliability Testing			07/Mar/21	12/Mar/21	6		
	dMFEA analysis			07/Mar/21	14/Mar/21	8		
	Implement dMFEA			14/Mar/21	05/Apr/21	23		
	Nice Finish (SEL shop ~2 wk lead)			05/Apr/21	15/Apr/21	11		
	Reassamble			16/Apr/21	23/Apr/21	8		
	Expo scramble			23/Apr/21	01/May/21	9		
	Technical Presentation			15/Apr/21	01/May/21	17		
	Display Poster			15/Apr/21	01/May/21	17		
	Design Expo			01/May/21	01/May/21	1		
	Final Course Deliverables			01/May/21	08/May/21	8		



Run #	X_known	Y_known	Z_known	L (in)	D (in)	Thic knes s	thet a (deg)	F1 (g)	F2 (g)	F3 (g)	F1' (g)	F2' (g)	F3' (g)
1	2	12	0,75	14	15,9	0,38	34	167	639	133	183	681	78
2	2	6	0,75	14	15,9	0,38	34	516	283	140	555	317	72
3	8	8	0,75	14	15,9	0,38	34	200	202	540	244	246	453
4	8	10	0,75	14	15,9	0,38	34	82	324	536	119	375	449
5	12	8	0,75	14	15,9	0,38	34	66	67	811	118	122	704
6	11	8	0,75	14	15,9	0,38	34	101	98	745	142	154	644
7	1	12	0,75	14	15,9	0,38	34	203	675	65	214	712	14
8	5	8	0,75	14	15,9	0,38	34	303	303	339	338	343	260
9	11	10	0,75	14	15,9	0,38	34	-19	216	746	16	277	650
10	3	12	0,75	14	15,9	0,38	34	131	607	200	113	618	210
11	9	10	0,75	14	15,9	0,38	34	51	286	603	85	344	517
12	6	10	0,75	14	15,9	0,38	34	151	384	406	182	438	324
13	5	10	0,75	14	15,9	0,38	34	184	421	336	211	471	262
14	11	6	0,75	14	15,9	0,38	34	221	-26	747	279	30	637
15	2	2	0,75	14	15,9	0,38	34	752	46	144	802	75	68
16	10	8	0,75	14	15,9	0,38	34	137	125	679	180	194	569
17	7	4	0,75	14	15,9	0,38	34	469	-7	480	533	37	378
18	3	14	0,75	14	15,9	0,38	34	12	727	200	31	784	133
19	4	14	0,75	14	15,9	0,38	34	-20	695	265	-6	757	197
20	6	4	0,75	14	15,9	0,38	34	499	33	410	565	67	316
21	10	6	0,75	14	15,9	0,38	34	252	8	680	307	68	571
22	1	14	0,75	14	15,9	0,38	34	80	793	65	98	834	14
23	6	8	0,75	14	15,9	0,38	34	270	266	406	305	318	323
24	4	2	0,75	14	15,9	0,38	34	685	-20	278	747	7	192
25	1	6	0,75	14	15,9	0,38	34	552	318	72	590	348	10

26	7	6	0,75	14	15,9	0,38	34	352	113	476	403	158	387
27	3	6	0,75	14	15,9	0,38	34	483	251	207	529	288	129
28	13	8	0,75	14	15,9	0,38	34	33	28	881	82	102	763
29	5	4	0,75	14	15,9	0,38	34	533	65	345	591	100	255
30	1	2	0,75	14	15,9	0,38	34	782	86	74	834	104	7
31	1	10	0,75	14	15,9	0,38	34	314	559	67	341	589	14
32	4	10	0,75	14	15,9	0,38	34	217	454	269	247	499	198
33	2	8	0,75	14	15,9	0,38	34	402	403	136	433	433	78
34	6	6	0,75	14	15,9	0,38	34	387	147	409	436	193	317
35	3	10	0,75	14	15,9	0,38	34	251	486	203	276	531	137
36	8	6	0,75	14	15,9	0,38	34	322	77	543	370	132	444
37	5	12	0,75	14	15,9	0,38	34	68	541	334	88	595	261
38	5	6	0,75	14	15,9	0,38	34	419	182	340	460	225	261
39	2	10	0,75	14	15,9	0,38	34	284	522	134	306	558	79
40	3	8	0,75	14	15,9	0,38	34	370	366	205	400	406	138
41	9	6	0,75	14	15,9	0,38	34	286	47	609	341	98	507
42	4	6	0,75	14	15,9	0,38	34	453	216	273	493	251	200
43	1	4	0,75	14	15,9	0,38	34	670	200	71	705	224	16
44	3	4	0,75	14	15,9	0,38	34	602	133	207	651	162	134
45	9	8	0,75	14	15,9	0,38	34	170	163	608	214	225	507
46	7	12	0,75	14	15,9	0,38	34	0	472	469	26	532	387
47	4	12	0,75	14	15,9	0,38	34	99	576	266	117	619	206
48	1	8	0,75	14	15,9	0,38	34	436	437	68	462	466	16
49	3	2	0,75	14	15,9	0,38	34	718	14	209	776	39	131
50	2	4	0,75	14	15,9	0,38	34	636	167	140	680	189	76
51	4	4	0,75	14	15,9	0,38	34	569	99	275	619	128	199
52	2	14	0,75	14	15,9	0,38	34	46	763	130	63	801	79
53	10	10	0,75	14	15,9	0,38	34	16	247	679	51	312	582
54	7	8	0,75	14	15,9	0,38	34	236	234	473	275	280	391
55	4	8	0,75	14	15,9	0,38	34	336	336	270	368	373	203
56	6	12	0,75	14	15,9	0,38	34	31	509	401	57	558	329
57	7	10	0,75	14	15,9	0,38	34	117	352	474	147	405	392

STATICS DERIVATIONS



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✧ Preferably make $B = \frac{D}{2}$

FBD: xz

$$\sum F_z: (F_1 + F_2) + F_3 - w = 0$$

$$w = F_1 + F_2 + F_3$$

$$\sum M_z: (F_1 + F_2)L - w(L - x_{cg}) = 0$$

$$x_{cg} = \frac{L(w - F_1 - F_2)}{w}$$

$$x_{cg} = \frac{L F_3}{w}$$

FBD: yz

$$\sum M_y: -w y_{cg} + F_3 \frac{D}{2} + F_2 D = 0$$

$$w y_{cg} = 0.5 F_3 D + F_2 D$$

$$y_{cg} = \frac{D(F_3 + 2F_2)}{2w}$$

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SIGNATURE _____ DATE _____

DISCLOSED TO AND UNDERSTOOD BY _____ DATE _____ PROPRIETARY INFORMATION

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FBD: xz tilted

$$\sum F_z: (F_1' + F_2') \cos(\theta) + w' \cos(\theta) + F_3' \cos(\theta) = 0$$

$$w' = F_1' + F_2' + F_3'$$

$$\sum M_1: w'_z x_{cg} - w'_x z_{cg} - F_3'_z = 0$$

$$w'_z = w' \cos(\theta)$$

$$w'_x z_{cg} = w'_z x_{cg} - F_3'_z L$$

$$w'_x = w' \sin(\theta)$$

$$z_{cg} = \frac{w'_z x_{cg} - F_3'_z L}{w'_x}$$

$$F_3'_z = F_3' \cos(\theta)$$

$$z_{cg} = \frac{w' \cos(\theta) x_{cg} - F_3' \cos(\theta) L}{w' \sin(\theta)}$$

$$z_{cg} = \frac{\cot(\theta) (w' x_{cg} - F_3' L)}{w'}$$

Continued to Page

SIGNATURE _____ DATE _____

DISCLOSED TO AND UNDERSTOOD BY _____ DATE _____ PROPRIETARY INFORMATION

VECTOR LINE DERIVATION



Define $L(t)$ as the vector line in 3-D space that goes through the Center of gravity

$L(t) = R^* + t \cdot b \cdot \hat{R}$
 $b = \text{thickness}$

$R^* = \frac{-1}{\omega} \hat{R} \times (\hat{i} \times \hat{F}_1 + \hat{i} \times \hat{F}_2 + \hat{i} \times \hat{F}_3)$

$\hat{F}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$
 $\hat{F}_2 = 0\hat{i} + D\hat{j} + 0\hat{k}$
 $\hat{F}_3 = L\hat{i} + \frac{1}{2}D\hat{j} + 0\hat{k}$

$\hat{F}_1 \times \hat{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$

$\hat{F}_2 \times \hat{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & D & 0 \\ 0 & 0 & F_{2z} \end{vmatrix} = DF_{2z} \hat{i}$

$\hat{F}_3 \times \hat{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ L & \frac{1}{2}D & 0 \\ 0 & 0 & F_{3z} \end{vmatrix} = \frac{D}{2}F_{3z} \hat{i} - LF_{3z} \hat{j}$

$R^* = \frac{-1}{\omega} \hat{R} \times \left(\frac{1}{2}D(F_3 + 2F_2) \hat{i} - LF_3 \hat{j} \right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \frac{\hat{R}}{\omega} \\ \frac{1}{2}D(F_3 + 2F_2) & -LF_3 & 0 \end{vmatrix} = \frac{-LF_3}{\omega} \hat{i} - \frac{D(F_3 + 2F_2)}{2\omega} \hat{j} + 0 \hat{k}$

$L(t) = \frac{-LF_3}{\omega} \hat{i} - \frac{D(F_3 + 2F_2)}{2\omega} \hat{j} + t \cdot b \cdot \hat{R}$

$R^* = \frac{-1}{\omega} \hat{R} \times (\hat{i} \times \hat{F}_1 + \hat{i} \times \hat{F}_2 + \hat{i} \times \hat{F}_3)$

$\hat{F}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$
 $\hat{F}_2 = 0\hat{i} + D\hat{j} + 0\hat{k}$
 $\hat{F}_3 = L \cos \theta \hat{i} + \frac{1}{2}D \hat{j} + L \sin \theta \hat{k}$
 $\hat{F}_1 = 0\hat{i} + 0\hat{j} + F_{1z} \hat{k}$
 $\hat{F}_2 = 0\hat{i} + 0\hat{j} + F_{2z} \hat{k}$
 $\hat{F}_3 = 0\hat{i} + 0\hat{j} + F_{3z} \hat{k}$

$\hat{F}_1 \times \hat{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0 \\ 0 & 0 & F_{1z} \end{vmatrix} = 0$

$\hat{F}_2 \times \hat{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & D & 0 \\ 0 & 0 & F_{2z} \end{vmatrix} = DF_{2z} \hat{i}$

$\hat{F}_3 \times \hat{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ L \cos \theta & \frac{1}{2}D & L \sin \theta \\ 0 & 0 & F_{3z} \end{vmatrix} = \frac{1}{2}DF_{3z} \hat{i} - L \cos \theta F_{3z} \hat{j}$

$R^* = \frac{-1}{\omega} \hat{R} \times \left(\frac{1}{2}D(F_3 + 2F_2) \hat{i} - LF_3 \hat{j} \right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \frac{\hat{R}}{\omega} \\ \frac{1}{2}D(F_3 + 2F_2) & -LF_3 & 0 \end{vmatrix} = \frac{-L \cos \theta F_3}{\omega} \hat{i} - \frac{D(F_3 + 2F_2)}{2\omega} \hat{j}$

$L(t) = \frac{-L \cos \theta F_3}{\omega} \hat{i} - \frac{D(F_3 + 2F_2)}{2\omega} \hat{j} + t \cdot \hat{R}$

[T] about y-axis $[T] \cdot L(t) = \begin{vmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{vmatrix} \begin{vmatrix} \frac{-L \cos \theta F_3}{\omega} \\ -\frac{D(F_3 + 2F_2)}{2\omega} \\ t \end{vmatrix}$

$L_y(t) = \frac{-L \cos^2 \theta F_3}{\omega} + t \sin \theta \hat{i} - \frac{D(F_3 + 2F_2)}{2\omega} \hat{j} + \frac{L \cos \theta \sin \theta F_3}{\omega} + t \cos \theta \hat{k}$

\hat{z} center of gravity will be at $L(t) = L_y(t)$

\hat{i} -component

$\frac{-LF_3}{\omega} = \frac{-L \cos^2 \theta F_3}{\omega} + t \sin \theta$

$t \sin \theta = \frac{-LF_3}{\omega} - \frac{L \cos^2 \theta F_3}{\omega}$

$t = \frac{-LF_3}{\omega \sin \theta} - \frac{L \cos^2 \theta F_3}{\omega \sin \theta}$

\hat{k} -component

$t = \frac{L \cos \theta \sin \theta F_3}{\omega} + t \cos \theta$

$t = \frac{L \cos \theta \sin \theta F_3}{\omega} - L \cos \theta \left(\frac{F_3}{\omega \sin \theta} - \frac{\cos^2 \theta F_3}{\omega \sin \theta} \right)$

$L = 14 \text{ in}$
 $\theta = 15.01^\circ$
 $F_3 = 172 \text{ g}$
 $\omega = 939 \text{ g}$
 $\omega = 941 \text{ g}$
 $F_2 = 204 \text{ g}$

$t = -1.755$

note needs to be positive like x and y components

PROTOTYPE SOURCES OF ERROR

- Limited accuracy with the kitchen scales
- Deflection in cardboard surface
- Nails not perfectly perpendicular to the scales
- Measurements not consistently taken from one location on the scales (i.e., not always in the center)
- Testing table not perfectly level

X AND Y UNCERTAINTIES

	L (in)	F_1 (g)	F_2 (g)	F_3 (g)	
Variable mean value	14	300	300	300	
Variable uncertainty	0.1	1	1	1	$w = F_1 + F_2 + F_3$
Uncertainty propagated to Xcg	0.03333333	-0.00518519	-0.005185	0.01037	
Total uncertainty in Xcg	0.0356711				$X_{cg} = \frac{L * F_3}{w}$

	D (in)	F_1 (g)	F_2 (g)	F_3 (g)	
Variable mean value	14	300	300	300	
Variable uncertainty	0.1	1	1	1	$w = F_1 + F_2 + F_3$
Uncertainty propagated to Ycg	0.1	-0.01556	0.015556	0	
Total uncertainty in Ycg	0.102391				$Y_{cg} = \frac{D(F_3 + 2F_2)}{2w}$

	L (in)	F_1' (g)	F_2' (g)	F_3' (g)	θ (deg)	Xcg (in)	t (in)
Variable mean value	14	300	300	300	15	6	0.375
Variable uncertainty	0.1	1	1	1	0.5	0.013	0.05
Uncertainty propagated to Zcg	-0.1244	0.019351	0.019351	-0.0387	-9.95214	0.048517	-0.05
Total uncertainty in Zcg	0.65327						