BAND-BEESTEN OMNI-BALL TESTING

Senior Lab Project Report

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Abstract

The Band-Beesten team has been tasked with creating a new mobile drumming platform for the UI Marching Band. One of the biggest problems with the previous design was the wheel design, as, even when the wheels were motorized, it was still difficult to move. Our team's goal is to increase the mobility of the Band-Beesten by improving the performance of the Omni-directional wheels that it utilizes. After a period of research, several designs were decided upon which consisted mostly of a small aluminum frame with Omni-wheels at varying angles and orientations. A testing apparatus was then constructed to simulate how each of the designs would work. In order to find the optimal setup for the Omni-balls that the Beesten would use, each setup was tested for the amount of friction that exists between the rollers and the ball. The testing was completed and the data was analyzed using a variety of tests such as several one-way ANOVA tests, a two-tailed T-Test, and a factorial ANOVA test to determine which of the variables being tested was the most statistically significant. Analysis of the setups showed that there were no statistically significant setups to use; therefore, the choice of setup came down to stability of the frame and general friction data that was found during testing. The final setup chosen ended up being the vertically oriented double Omni-wheels at 50 degrees from the equator of the ball which worked very well and allowed any user to push the Beesten in any direction easier than the previous model.

Introduction

Background:

The Band-Beesten is a mobile drumming platform built for the UI Marching Band that is designed to hold no less than five instruments and move easily on both football turf and a basketball court. One of the biggest problems with the previous incarnation of the Band-Beesten was mobility. The back two wheels were basketballs rolling on ball casters, while the front wheel used motorized Omni-wheels to help the user move the entire frame. With this set-up, the back wheels had far too much friction both on the ground and in the ball casters to overcome, which made the whole platform sluggish and difficult to move. The introduction of the motor on the front wheel helped alleviate some of this, but it still had difficulty moving swiftly enough for the marching band to do what they had envisioned. A redesign of the wheels was clearly the first step in a complete redesign of the Band-Beesten. Once the wheels actually allowed for a greater range of free movement, a frame could be engineered that could hold all the instruments required by the marching band.

Motivation:

One of the biggest problems with the previous design was the wheel design, as even when the wheels were motorized it was still difficult to move. In order to move on to designing the frame, we needed to understand how to setup the wheels. The idea of a friction reduced Omni-ball came from several Ball-Bot designs where a robot is supported by a ball that is rotated via three or more Omni-wheels [1]. Such Ball-Bots have been a relatively recent breakthrough in robotics as the motorized Omni-wheels that keep the robot stable have only recently been created. Our wheel designs borrow the basic structure of the Ball-Bot Omni-ball setup, although ours has been simplified somewhat due to not motorizing it. The purpose of this experiment was to determine the best configurations of wheels using both Omni-wheels and ball casters. Once the testing was completed, we created a finalized wheel design that can be used effectively and meets all of our design requirements

Objective:

The wheel placement as well as how the wheels will be oriented all play major roles in determining how easily the ball will roll and the amount of friction to get the ball moving. Therefore, the problem is which orientation and angle will minimize the friction while still maximizing the range of movement. We tested the dependent variable of friction by varying independent variables of wheel angle and wheel placement. The angle levels will be 40, 45, 50 and 55 degrees and wheel placement levels will be single Omni-wheels horizontal, single Omni-wheels vertical, double Omni-wheels horizontal, double Omni-wheels vertical, and a control of ball casters. The data was analyzed using several one-way ANOVA analyses to determine individual interactions as well as a multilevel factorial for determining interactions with the double wheels. Each statistical analysis was conducted to 95% certainty with alpha values of 0.05. The null hypothesis was that Omni-wheel type, angle, placement, and interactions between angle and placement have no effect on friction. The alternative hypothesis was that Omni-wheel type, angle, placement, and interactions between angle and placement do have an effect on friction.

Methods

The experiment we conducted was designed to give us quantitative data on the friction coefficients for each of the different setups. In order to run the tests that would give us the friction data we needed, we first had to build a testing platform that allowed for varying angles and orientations to be tested using both types of wheels. After the testing, we analyzed the data to find the optimal setup for our final design.

Equipment Setup:

The platform needed to be able to roll on a ball with the Omni-wheels/caster at varying angles and allow us to read the amount of force required to start the movement of the ball. We designed the platform to have three curved arms with slots in each so that different attachments and arrangements could be connected. Figure 1 consists of renders of the idea; one with the Omni-wheels (left) and one with the ball casters (right).

Figure 1: Wheel/Caster Placement Apparatus

This singular apparatus allows for the testing of all crucial variables as well as suitably simulate the weight that the actual wheels would see once they were built and in use. The same basketball from the 2012 BandBeesten was used, but inflated to reach a very firm-to-the-touch pressure equaling about 8psi.

Upon completion of the testing apparatus, a frame was constructed to simulate both the weight of the frame complete with all necessary instruments as well as the balance that would come from having two more wheels. This apparatus can be seen in Figure 2.

Figure 2: Experimental Set-up

The frame worked by being pulled from four corners simultaneously by ropes that were all connected at the end to a fishing scale. The apparatus was first used on the ball casters to get a general force control for the system. The previous Band-Beesten wheels used ball casters to create an Omni-directional wheel, so we decided we should use the same setup as our benchmark.

Testing was done through the frame setup shown above tethered to a force gage. Our team was given an electronic fishing scale that reads the weight of an object in kilograms for our friction testing. We also acquired a Dataq 155 data acquisition device that could be linked to the scale and to a computer via USB.   The scale was connected to the four ropes attached to the wooden frame and pulled just until the entire apparatus moved. This force would then be sent to through the data-logger to our computer where the friction would be calculated using the known weights and found force. The basic setup is shown in Figure 3.

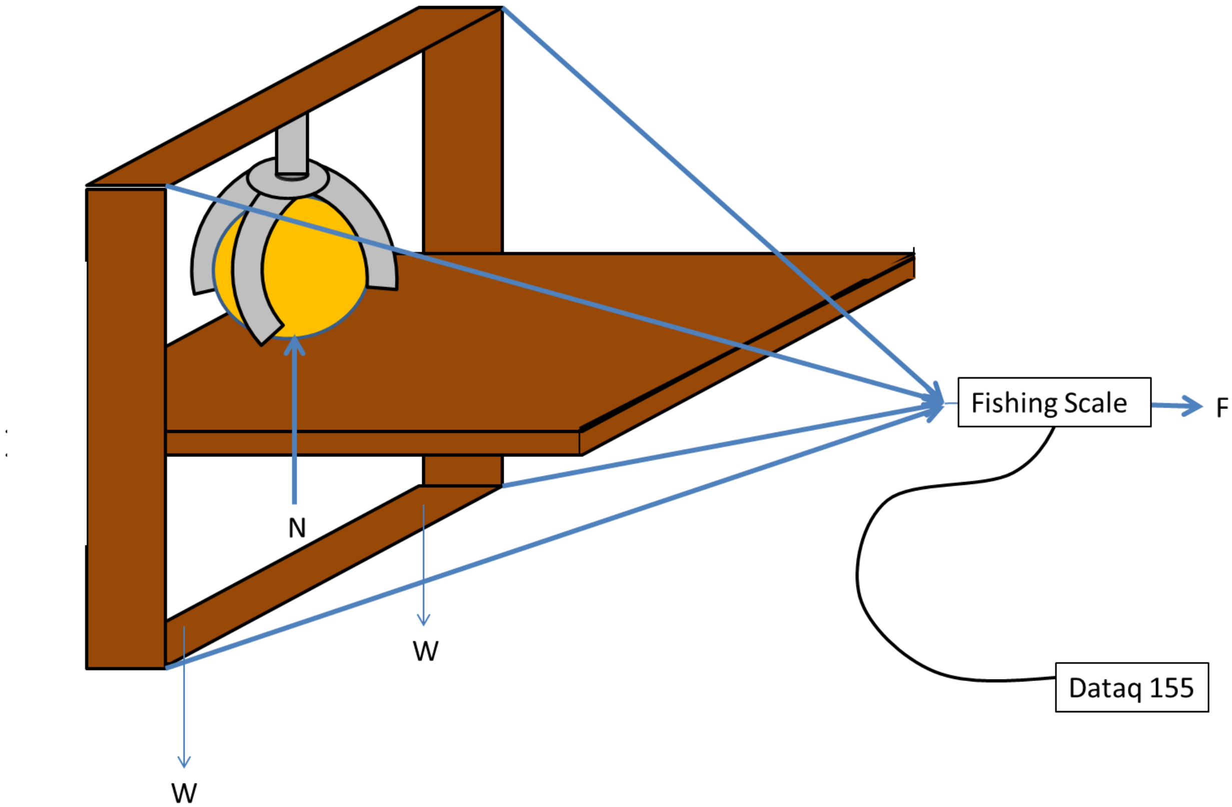


Figure 3: Testing Setup

All four lines are tethered to each corner of the wooden frame, which is supported by the testing apparatus that can glide along the table that it is on. The user pulls lightly on the scale until the ball moves, at which point the user stops pulling and then stops the Dataq to prevent any extraneous data from being collected. Extra weight can be placed onto the bottom of the wooden frame so that the apparatus can be tested with proper weight that would adequately simulate the weight of the Beesten frame and instruments.

Calibration:

The data that the Dataq obtained from the scale was not in terms of force, but instead registered any change in force as a minute change in voltage. As we needed force data in order to calculate the friction coefficients, the Dataq needed to be properly calibrated. Calibration was done by attaching weights to the scale and reading the voltage output as seen in Figure 4. A simple regression equation was then determined that output the force used. When the scale was properly calibrated to accurately show minute changes of force, the testing of the different set-ups could begin.

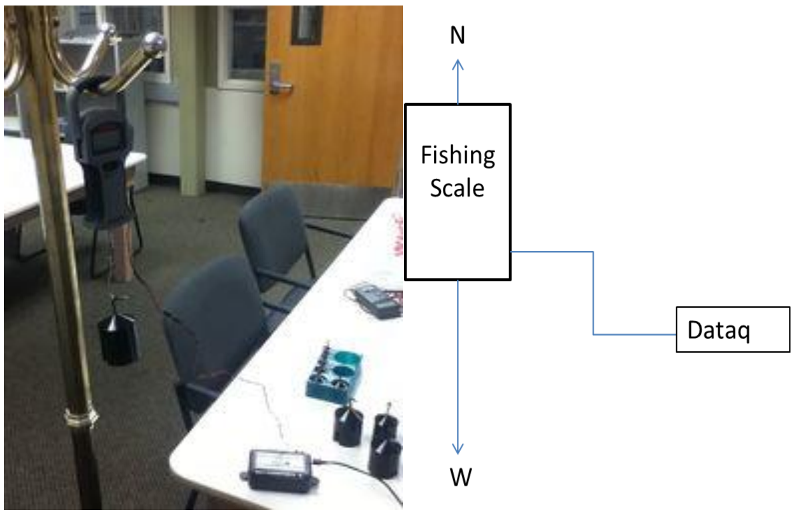


Figure 4: Calibration of Scale

The calibration data used can be found in Appendix A.

Mathematical Model:

Once we had at least three data points for each of the different configurations, we were able to solve for the coefficient of static friction between the ball and the omni-wheels, assuming that the friction between the ball and the table is negligible. To find the static friction coefficient for each set-up, one must know the normal force acting on the apparatus as well as the force to begin movement [2].

$$F_s = m_s \cdot N$$

The weight of the apparatus was known and can be actively changed by adding extra weights to the testing frame, thus allowing us to solve for the normal force acting on it. The force to begin movement was calculated using the fishing scale/data acquisition device as well as the correlation found through careful testing between the voltage output of the Dataq and actual force.

Experimental Design:

Due to the large amount of variables that this testing had to deal with and how some of the variables were only available for specific setups, we needed to perform several individual analyses in order to determine how each setup compared to the others. Each of the roller options were individually analyzed in order to understand what was significant in each setup. The single Omni-wheel setups used one-way ANOVA to determine whether the angles were significant with the orientation, and then a two sample T-test was used to determine if orientation was significant. The double Omni-wheels used a factorial analysis with the friction as the dependent variable and wheel alignment and wheel position as the independent variables. The ball casters were analyzed as the control with another one-way ANOVA to determine if their angle was significant. The final analysis compared all the friction data for each of the setups to determine if the choice of roller was significant. The levels of the wheel alignment would include the four different possibilities of a single Omni-wheel at either vertical or horizontal, and two wheels at either vertical or horizontal. The levels of the wheel placement would be the four different angles of 40, 45, 50, and 55 degrees. For each combination of variable, we gathered at least three data points, giving a total of sixty data points. While running the analysis, the dependent variable friction coefficient was evaluated at a 95% confidence with an alpha value of 0.05. The final analysis was completed in Minitab v16 and it indicated which combined setup has the greatest influence on friction force [3].

Experimental Procedure:

After calibration was complete on the scale, we began to test the different setups. The tests were conducted by pulling the entire apparatus with a constant force and measuring when the ball began to move. This proved difficult, however, as there was no easy way to insure that the entire frame was being pulled with the same force. This problem was eventually solved by attaching four ropes to each corner of the frame, as seen previously in Figure 3. This allowed us to pull on the end of the force gage until the entire apparatus began to move without having to worry about unequal force distribution.

Results

After collecting the results from our testing, we then conducted our statistical analysis of the data. The raw test data used can be found in Appendix B and the complete Minitab file of each analysis can be found in Appendix C at the end of this document.

The first analysis we performed was on the single wheel data. We wanted to compare the orientation of the single wheels to the individual angles used for both orientation levels. All analyses were performed at 95% certainty with alpha values of 0.05 to maintain homogeneity. We started with the vertical orientation and we ran a one-way ANOVA to determine if the angles are statistically significant. Minitab was used to determine the interactions as shown in Figure 5.

```
Source  DF        SS        MS     F      P
Factor   2  0.002518  0.001259  2.24  0.188
Error    6  0.003378  0.000563
Total    8  0.005896

S = 0.02373   R-Sq = 42.71%   R-Sq(adj) = 23.61%


                              Individual 95% CIs For Mean Based on
                              Pooled StDev
Level  N     Mean    StDev    ---+---------+---------+---------+------
40     3  0.21200  0.01684                (-----------*----------)
45     3  0.17591  0.02171    (----------*----------)
50     3  0.17716  0.03056     (----------*----------)
                              ---+---------+---------+---------+------
                               0.150     0.180     0.210     0.240

Pooled StDev = 0.02373


Grouping Information Using Tukey Method

    N     Mean  Grouping
40  3  0.21200  A
50  3  0.17716  A
45  3  0.17591  A

Means that do not share a letter are significantly different.
```

Figure 5: Single Vertical Wheel One-Way ANOVA File

This data shows a P-value of 0.188 and each of the intervals overlap each other. This means that the angles are not statistically significant for the vertical orientation. The 40 degree level was somewhat off from the others, so we removed it to see how the 45 and 50 degree levels would interact alone with each other (Figure 6).

```
              40 subtracted from:
Lower     Center     Upper     -----+---------+---------+---------+----
45  -0.09555  -0.03609  0.02336  (--------------*--------------)
50  -0.09430  -0.03484  0.02461  (--------------*--------------)
                                 ----+---------+---------+---------+-----
                                   -0.080    -0.040     0.000     0.040
```

Figure 6: Single Vertical Wheel One-Way ANOVA File without 40 degree angle

This shows that there is no difference between the 45 and 50 degree levels, thus the angle is not statistically significant to the vertical orientation of one wheel.

The next analysis performed was for the horizontal orientation of one wheel. Figure 7 shows the ANOVA analysis for the horizontal orientation against the angle levels.

```
          Source  DF        SS         MS      F      P
Factor    2  0.007522  0.003761   7.74  0.022
Error     6  0.002915  0.000486
Total     8  0.010437

S = 0.02204   R-Sq = 72.07%   R-Sq(adj) = 62.76%


                           Individual 95% CIs For Mean Based on
                           Pooled StDev
Level  N     Mean     StDev  -----+---------+---------+---------+----
H45    3  0.21906  0.01914                 (--------*-------)
H50    3  0.20952  0.01634              (--------*--------)
H55    3  0.15352  0.02871  (--------*--------)
                            -----+---------+---------+---------+----
                               0.140     0.175     0.210     0.245

Pooled StDev = 0.02204


Grouping Information Using Tukey Method

      N     Mean  Grouping
H45  3  0.21906  A
H50  3  0.20952  A
H55  3  0.15352    B

Means that do not share a letter are significantly different.
```

Figure 7: Single Horizontal Wheel One-Way ANOVA File

Based on the analysis, the found P-value was 0.022, which is less than the alpha of 0.05, indicating that the angle is statistically significant for the vertical single wheel.

The next interaction analyzed was the two orientations for the single wheels using the mean values of each of the single wheel friction factors based on wheel orientation. This was done via a two sample T-Test in Minitab to determine if there was a significant difference between the two data sets shown in Figure 8.

```
Two-sample T for V vs H

   N    Mean   StDev  SE Mean
V  9  0.1884  0.0271   0.0090
H  9  0.1940  0.0361    0.012


Difference = mu (V) - mu (H)
Estimate for difference:  -0.0057
95% CI for difference:  (-0.0380, 0.0266)
T-Test of difference = 0 (vs not =): T-Value = -0.38  P-Value = 0.712
DF = 14
```

Figure 8: Two-Sample T-Test for Comparing Single Wheel Orientations

This test showed a P-value of 0.712 which is much higher than the alpha value of 0.05 for this test. This means that we failed to reject the null hypothesis of the two means being the same, meaning that the orientation is not statistically significant for the single wheels.

The next tests we ran were to determine what was significant with the double wheel setups. For this analysis, we used a factorial ANOVA shown in Figure 9 with angle and orientation as factors compared to the friction.

General Linear Model: Friction versus Angle, Orientation

Factor	Type	Levels	Values
Angle	fixed	4	40, 45, 50, 55
Orientation	fixed	2	Horiz., Vert.

Analysis of Variance for Friction, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Angle	3	0.0132337	0.0132337	0.0044112	11.00	0.000
Orientation	1	0.0066035	0.0066035	0.0066035	16.47	0.001
Angle*Orientation	3	0.0030742	0.0030742	0.0010247	2.56	0.092
Error	16	0.0064162	0.0064162	0.0004010		
Total	23	0.0293276				

S = 0.0200253   R-Sq = 78.12%   R-Sq(adj) = 68.55%

Figure 9: Factorial ANOVA File for Double Wheels

This analysis compared the angle, orientation, and the interaction between angle and orientation to determine what had a significant effect on the result. Based on the results found, the angle was found to be statistically significant with a P-value of less than 0.0001. Orientation was found to be statistically significant with a P-value of 0.001. The interaction effects were found to not be statistically significant with a P-value of 0.092. This shows that the only non-significant factor for the double wheels was the interaction effect of the angle and orientation.

The next analysis performed was on the control of the ball casters to determine if the different angles are significant contributors to the friction. This was done by a one-way ANOVA analysis to determine if any of the levels are significant (Figure 10).

```
Source  DF       SS       MS     F      P
Factor   2  0.00340  0.00170  1.38  0.321
Error    6  0.00739  0.00123
Total    8  0.01079

S = 0.03509   R-Sq = 31.53%   R-Sq(adj) = 8.71%


                               Individual 95% CIs For Mean Based on
                               Pooled StDev
Level  N     Mean    StDev     -------+---------+---------+---------+--
45     3  0.18880  0.02842                  (-----------*-----------)
50     3  0.17141  0.05257             (-----------*-----------)
55     3  0.14171  0.01109     (-----------*-----------)
                               -------+---------+---------+---------+--
                                   0.120     0.160     0.200     0.240

Pooled StDev = 0.03509


Grouping Information Using Tukey Method

    N     Mean  Grouping
45  3  0.18880  A
50  3  0.17141  A
55  3  0.14171  A
```

Figure 10: Ball Casters One-Way ANOVA File

This analysis shows a P-value of 0.321 meaning that the angles are not statistically significant for the ball caster's friction amounts.

The final analysis performed was a one-way ANOVA on the general friction factors for the three different types of roller: the single wheel, the double wheel, and the ball casters. To do this, all the friction data was reconciled together under their respective rollers. The Minitab file for this analysis is shown below in Figure 11.

```
Source  DF       SS       MS     F      P
Factor   2  0.00509  0.00255  2.16  0.126
Error   48  0.05660  0.00118
Total   50  0.06169

S = 0.03434   R-Sq = 8.26%   R-Sq(adj) = 4.43%
```

Figure 11: Roller Choice One-Way ANOVA File

This test returned a P-value of 0.126, meaning that the roller choice is not statistically significant for choosing the optimum setup.

Discussion

The analysis of the several different data sets proved to be quite insightful in determining what wheel setup would bring about the least amount of friction. Several conclusions can be drawn from the analyses performed as well as from the actual experience of performing the tests.

The single wheel analysis showed that the angle is not significant for the vertical setup, while the angle is significant for the horizontal setup. This was expected, as the vertical orientation should not really affect how easily the ball will roll. No matter the angle used, the vertical wheel should perform nearly identically in terms of friction as its main bearing allows it to easily rotate vertically with the ball. The horizontal orientation, however, is much more sensitive to the angle that it is in contact with as the main mode of rotating the ball vertically is the smaller bearings on its outside ring, which tend to impede rotating in the vertical direction more. When we discovered that the angle only really makes a difference for the horizontal, we decided to check how the orientations compare without the clarifying angles. The data was compiled for each orientation and the resulting analysis showed that the orientation used was not a statistically significant contributor to the friction.

The next step to finding the optimum setup was to test how the double wheel factors contributed to the overall friction found. This was done through a Factorial ANOVA to determine how each of the factors played in to determining the friction coefficient. The test was run with the four different angles of 40, 45, 50, and 55 and the two different orientations of vertical and horizontal. The results of the analysis showed that both the angle and orientation are statistically significant on their own, however, the interactions between the two were not statistically significant. This told us that we could change the setup for the doubles to whatever we wanted (within the confines of our testing) to find the setup with the least amount of friction and we wouldn't need to worry about any orientation combination that would make a large statistical difference.

The next analysis that we ran was on the ball casters. The ball casters were the original roller setup for the previous model of the Band-Beesten and thus we felt it necessary to include with our data as a sort of control variable when trying to find the best new setup. The only factor we could change for the ball casters was the angle; therefore, we ran a one-way ANOVA on the three different angles that the ball casters could reach to see if there was any sort of statistical difference between them. The P-value of 0.321 showed that the angle choice was not statistically significant, so we could simply use the data we had as a comparison for the new setups.

The last analysis we performed was to determine if the choice of roller we used was a significant variable. For this, we combined all the friction data for each roller choice and ran a one-way ANOVA. The result showed a P-value of 0.126 indicating that the roller choice wasn't statistically significant in determining friction factors. The data from the analysis was not overwhelmingly useful. Much of the data was found to not be significant. This could have been due to the user error in performing the experiment. The way we set up the experiment was to have the same person pull on a rope with roughly the same force for every trial and mark when the ball began to move. This was not terribly accurate and was most likely why not much of the data is significant. It could also have been that there simply was no statistical difference between the different setups.

As few of the variables we tested were statistically significant, we were able to plot the data and find the mean values of each setup to choose the most optimum. The plot and data is found in Appendix C. When the data is plotted, it appears that the best choice would be the double horizontal at 55 degrees. The problem with this setup was how unstable the ball was due to the height of the rollers. With too much motion, the ball has a tendency to slip out of the experiment setup, which we didn't want to risk with the final product. The final choice eventually came down to deciding what would be the qualitatively stable and still have a reasonable friction coefficient. Based on the single vertical analysis which stated that the angle is not statistically significant, the increased grip of the double wheels, and the wheel stability of the 45 degree angle, the final choice was made to be the double-vertical wheel at 45 degrees (Figure 12).

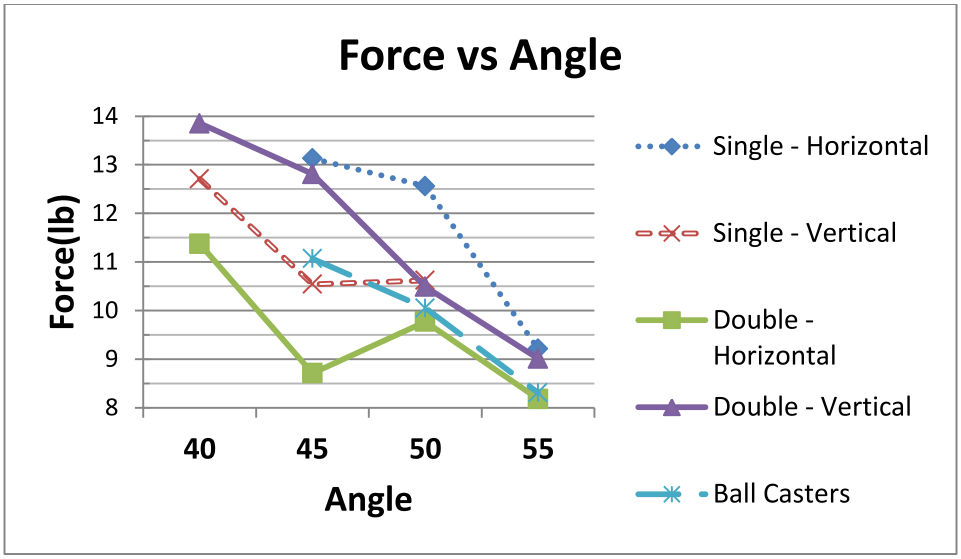


Figure 12: Force vs Angle Plot

Upon completion of the testing, our team went ahead with the construction of our Omni-ball design and completed a prototype Band-Beesten for the University of Idaho Marching Band to use during a football game halftime show. While the user found that the entire Beesten was rather heavy and found it difficult to push for that reason, the wheels worked very well on the turf and the drummer who used it was happy with the product. One large problem with the design was that it did not allow for any sized ball different from the one used in testing. Any larger balls would lock up the system, and smaller balls would slip out of the apparatus if they weren't over-pressurized. For the future, things such as the ball size and ball pressure should be taken into account when testing the friction forces of each setup.

Conclusions

The purpose of these tests and corresponding analyses was to determine what factors in each setup were statistically significant and from that to determine the optimum setup to use in the final Band-Beesten product. Each of the various setups had certain variables that we could change including the angle of the rollers, the orientation of the rollers, and the type of rollers.

We needed to know if any of the individual or combinations of variables would make a large statistical difference for our data. If any of them did, we could isolate the variable more closely and determine if it could help us better reach our goal of a low amount of friction between the ball and the rollers. Analyses of the single Omni-wheel rollers at 95% confidence showed us that the angle is only statistically significant if the roller is in a horizontal position. When we compared the two orientations of horizontal and vertical however, it was found that the orientation is not statistically significant, therefore we could assume that the angle is not statistically significant for the single rollers. The analysis on the double rollers showed that both angle and orientation are statistically significant, while the interaction effects between them are not statistically significant. This meant that while both the angle and orientation mattered, how the two interacted did not. Finally, we wanted to have a more general picture of if the different roller choices really mattered. The ANOVA analysis of this data showed that the roller choice didn’t really matter either. We then decided to pick the setup that had the most stability and that kept the ball from slipping out which turned out to be the vertical-two wheel setup at 45 degrees.

References

[1] Swiss Federal Institute of Technology Zurich, "Modeling and Control of a Ballbot," *Swiss Federal Institute of Technology,* 27-40, 2010. [Online]. Avaliable: http://students.asl.ethz.ch/upl_ pdf/214-report.pdf. [Accessed Oct. 1,2013].

[2] R.C. Hibbler, *Engineering Mechanics: Statics*, 12th ed. Upper Saddle River, NJ: Pearson Prentice Hall, 2010.

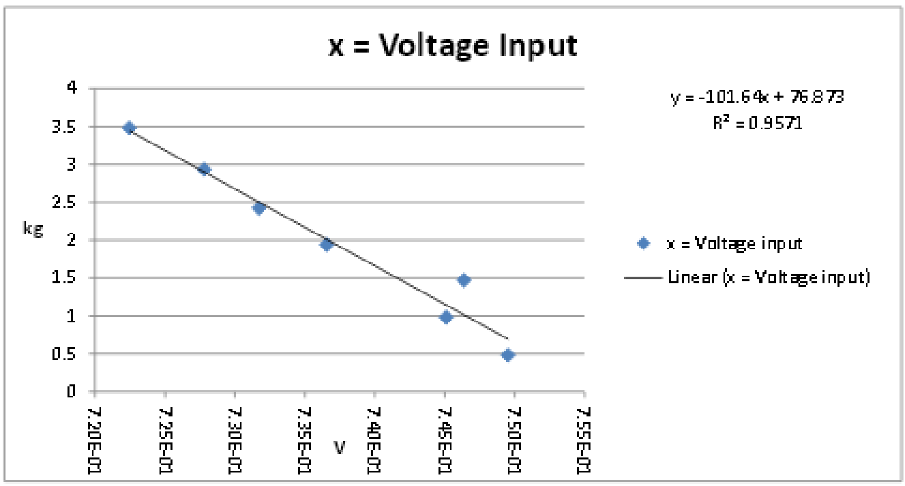
[3] T.C. Urdan, Statistics in Plain English, 3rd ed. New York, NY: Taylor and Francis Group LLC, 2010

Appendices

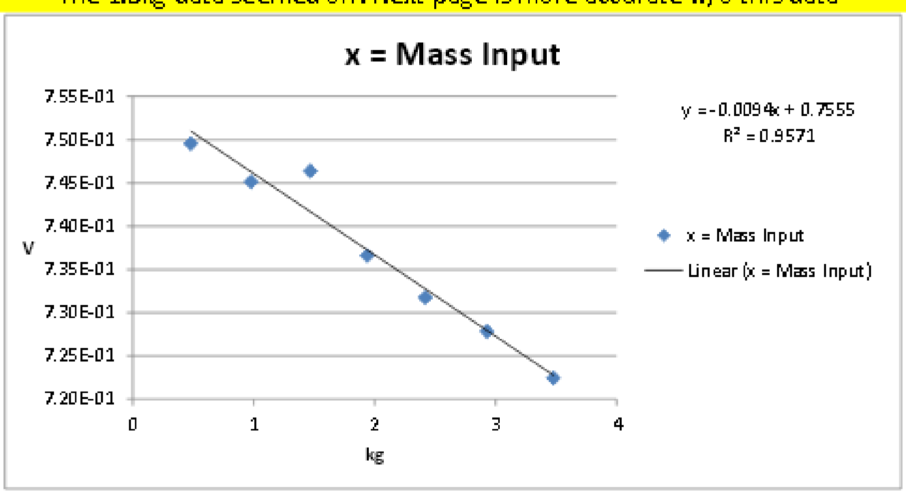
Appendix A: Fishing Scale Calibration Data

Fish Scale Conversion Test w/1.5 kg data

Expected Mass (kg)	Measured Mass (kg)	Baseline (V)	W/mass (V)
0.5	0.48	7.5409E-01	7.4954E-01
1	0.98	7.5421E-01	7.4511E-01
1.5	1.47	7.5416E-01	7.4638E-01
2	1.94	7.5384E-01	7.3657E-01
2.5	2.42	7.4505E-01	7.3173E-01
3	2.93	7.5503E-01	7.2781E-01
3.5	3.48	7.5404E-01	7.2245E-01
	Baseline Average	7.5292E-01	

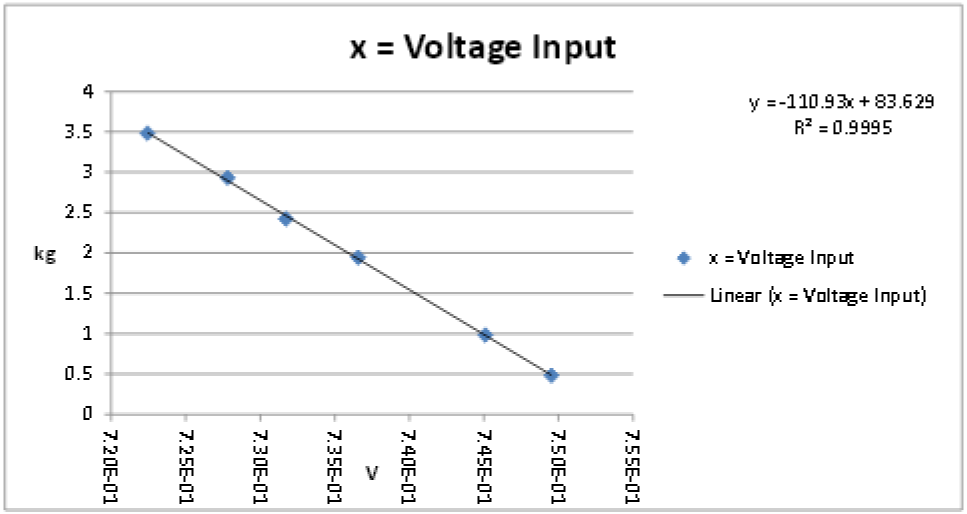


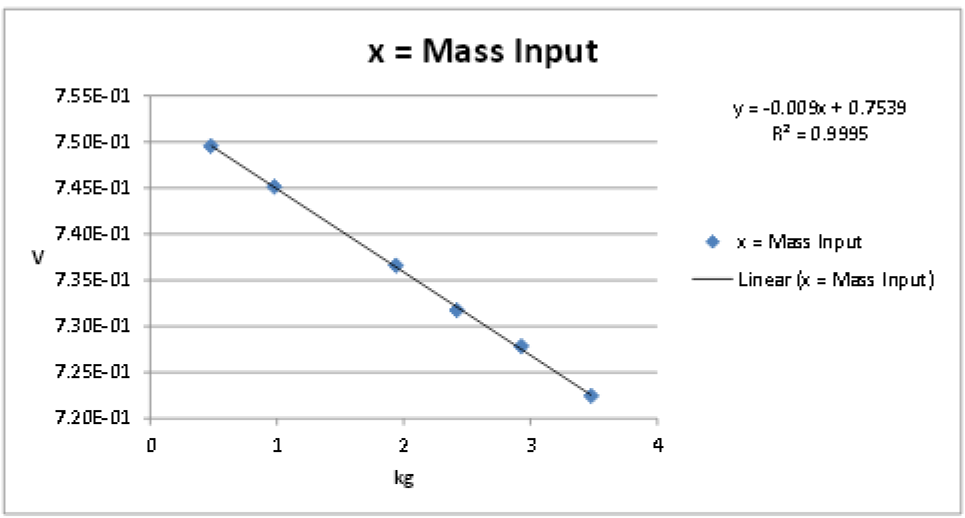
The 1.5kg data seemed off. Next page is more accurate w/o this data



Fish Scale Conversion Test w/o 1.5kg data

Expected Mass (kg)	Measured Mass (kg)	Baseline (V)	W/mass (V)
0.5	0.48	7.5409E-01	7.4954E-01
1	0.98	7.5421E-01	7.4511E-01
2	1.94	7.5384E-01	7.3657E-01
2.5	2.42	7.4505E-01	7.3173E-01
3	2.93	7.5503E-01	7.2781E-01
3.5	3.48	7.5404E-01	7.2245E-01
	Baseline Average	7.5271E-01	





Appendix B: Raw Test Data

Single Wheel-Vertical Data:

Using the correlation w/o 1.5 kg: y=-110.93x+83.629 to find the Force, y.

F=mu*N

				F		N		mu
Angle	**Run #**	**Time (s)**	**Voltage (V)**	**Force (kg)**	**Force (lb)**	**Weight (kg)**	**Weight (lb)**	**Friction Coefficient**
40	1	1.008	6.9794E-01	6.2065	13.6830	27.2	59.85	0.2282
	2	0.7	7.0618E-01	5.2925	11.6678	27.2	59.85	0.1946
	3	0.854	7.0160E-01	5.8005	12.7879	27.2	59.85	0.2133
	Average	-	0.701907	5.7665	12.7129	-	-	0.2120

				F		N		mu
Angle	**Run #**	**Time (s)**	**Voltage (V)**	**Force (kg)**	**Force (lb)**	**Weight (kg)**	**Weight (lb)**	**Friction Coefficient**
45	1	0.487	7.1442E-01	4.3784	9.6527	27.2	59.85	0.1610
	2	0.621	7.1320E-01	4.5137	9.9510	27.2	59.85	0.1659
	3	0.808	7.0465E-01	5.4622	12.0420	27.2	59.85	0.2008
	Average	-	0.710757	4.7848	10.5486	-	-	0.1759

				F		N		mu
Angle	**Run #**	**Time (s)**	**Voltage (V)**	**Force (kg)**	**Force (lb)**	**Weight (kg)**	**Weight (lb)**	**Friction Coefficient**
50	1	0.804	7.1228E-01	4.6158	10.1760	27.2	59.85	0.1697
	2	0.592	7.1686E-01	4.1077	9.0560	27.2	59.85	0.1510
	3	1.054	7.0221E-01	5.7328	12.6387	27.2	59.85	0.2108
	Average	-	0.710450	4.8188	10.6236	-	-	0.1772

				F		N		mu
Angle	**Run #**	**Time (s)**	**Voltage (V)**	**Force (kg)**	**Force (lb)**	**Weight (kg)**	**Weight (lb)**	**Friction Coefficient**
55	1	Does		83.6290	184.3702	27.2	59.85	3.0746
	2	not		83.6290	184.3702	27.2	59.85	3.0746
	3	work		83.6290	184.3702	27.2	59.85	3.0746
	Average	!!	#DIV/0!	83.6290	184.3702	-	-	3.0746

Single Wheel-Horizontal Data:

Using the correlation w/o 1.5 kg: y=-110.93x+83.629 to find the Force, y.

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
40	1	Does		83.6290	184.3702	27.2	59.85	3.0746
	2	not		83.6290	184.3702	27.2	59.85	3.0746
	3	work		83.6290	184.3702	27.2	59.85	3.0746
	Average	!!	#DIV/0!	83.6290	184.3702	-	-	3.0746

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
45	1	1.038	7.0557E-01	5.3601	11.8170	27.2	59.85	0.1971
	2	0.804	6.9702E-01	6.3086	13.9080	27.2	59.85	0.2319
	3	1.158	6.9794E-01	6.2065	13.6830	27.2	59.85	0.2282
	Average	-	0.700177	5.9584	13.1360	-	-	0.2191

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
50	1	0.808	7.0313E-01	5.6308	12.4138	27.2	59.85	0.2070
	2	0.921	6.9824E-01	6.1732	13.6096	27.2	59.85	0.2270
	3	0.704	7.0618E-01	5.2925	11.6678	27.2	59.85	0.1946
	Average	-	0.702517	5.6988	12.5637	-	-	0.2095

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
55	1	0.55	7.2357E-01	3.3634	7.4150	27.2	59.85	0.1237
	2	0.55	7.0953E-01	4.9208	10.8486	27.2	59.85	0.1809
	3	1.013	7.1564E-01	4.2431	9.3543	27.2	59.85	0.1560
	Average	-	0.716247	4.1758	9.2060	-	-	0.1535

Two Wheels-Vertical Data:

Using the correlation w/o 1.5 kg: y=-110.93x+83.629 to find the Force y.

F=mu*N

				F		N		mu
Angle	**Run #**	**Time (s)**	**Voltage (V)**	**Force (kg)**	**Force (lb)**	**Weight (kg)**	**Weight (lb)**	**Friction Coefficient**
40	1	2.35	0.703430	5.5975	12.3404	27.8	61.2	0.2013
	2	3.567	0.696110	6.4095	14.1306	27.8	61.2	0.2306
	3	1.417	0.692140	6.8499	15.1014	27.8	61.2	0.2464
	Average	-	0.697227	6.2856	13.8575	-	-	0.2261

				F		N		mu
Angle	**Run #**	**Time (s)**	**Voltage (V)**	**Force (kg)**	**Force (lb)**	**Weight (kg)**	**Weight (lb)**	**Friction Coefficient**
45	1	1.55	0.706480	5.2592	11.5945	27.8	61.2	0.1892
	2	2.3	0.701600	5.8005	12.7879	27.8	61.2	0.2087
	3	1.283	0.696410	6.3762	14.0572	27.8	61.2	0.2294
	Average	-	0.701497	5.8120	12.8132	-	-	0.2091

				F		N		mu
Angle	**Run #**	**Time (s)**	**Voltage (V)**	**Force (kg)**	**Force (lb)**	**Weight (kg)**	**Weight (lb)**	**Friction Coefficient**
50	1	2.117	0.706790	5.2248	11.5187	27.8	61.2	0.1879
	2	2.15	0.718690	3.9047	8.6084	27.8	61.2	0.1405
	3	1.917	0.707400	5.1571	11.3695	27.8	61.2	0.1855
	Average	-	0.710960	4.7622	10.4989	-	-	0.1713

				F		N		mu
Angle	**Run #**	**Time (s)**	**Voltage (V)**	**Force (kg)**	**Force (lb)**	**Weight (kg)**	**Weight (lb)**	**Friction Coefficient**
55	1	1.8	0.711060	4.7511	10.4744	27.8	61.2	0.1709
	2	2.538	0.720210	3.7361	8.2367	27.8	61.2	0.1344
	3	1.858	0.719910	3.7694	8.3101	27.8	61.2	0.1356
	Average	-	0.717060	4.0855	9.0071	-	-	0.1470

Two Wheels-Horizontal Data:

Using the correlation w/o 1.5 kg: y=-110.93x+83.629 to find the Force, y.

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
40	1	1.758	0.701600	5.8005	12.7879	27.8	61.2	0.2087
	2	1.867	0.709230	4.9541	10.9219	27.8	61.2	0.1782
	3	2.333	0.711360	4.7178	10.4010	27.8	61.2	0.1697
	Average	-	0.707397	5.1575	11.3703	-	-	0.1855

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
45	1	2.333	0.714110	4.4128	9.7285	27.8	61.2	0.1587
	2	1.821	0.716550	4.1421	9.1318	27.8	61.2	0.1490
	3	1.258	0.724180	3.2957	7.2658	27.8	61.2	0.1186
	Average	-	0.718280	3.9502	8.7087	-	-	0.1421

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
50	1	2.058	0.713810	4.4461	9.8019	27.8	61.2	0.1599
	2	1.871	0.713500	4.4804	9.8777	27.8	61.2	0.1612
	3	1.35	0.714420	4.3784	9.6527	27.8	61.2	0.1575
	Average	-	0.713910	4.4350	9.7774	-	-	0.1595

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
55	1	2.008	0.725100	3.1937	7.0408	27.8	61.2	0.1149
	2	2.492	0.718990	3.8714	8.5351	27.8	61.2	0.1393
	3	1.2	0.717160	4.0744	8.9826	27.8	61.2	0.1466
	Average	-	0.720417	3.7132	8.1861	-	-	0.1336

Two Wheels-Vertical Data:

Using the correlation w/o 1.5 kg: y=-110.93x+83.629 to find the Force y.

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
40	1	Does		83.6290	184.3702	26.6	58.6	3.1439
	2	not		83.6290	184.3702	26.6	58.6	3.1439
	3	work		83.6290	184.3702	26.6	58.6	3.1439
	Average	!!		83.6290	184.3702	-	-	3.1439

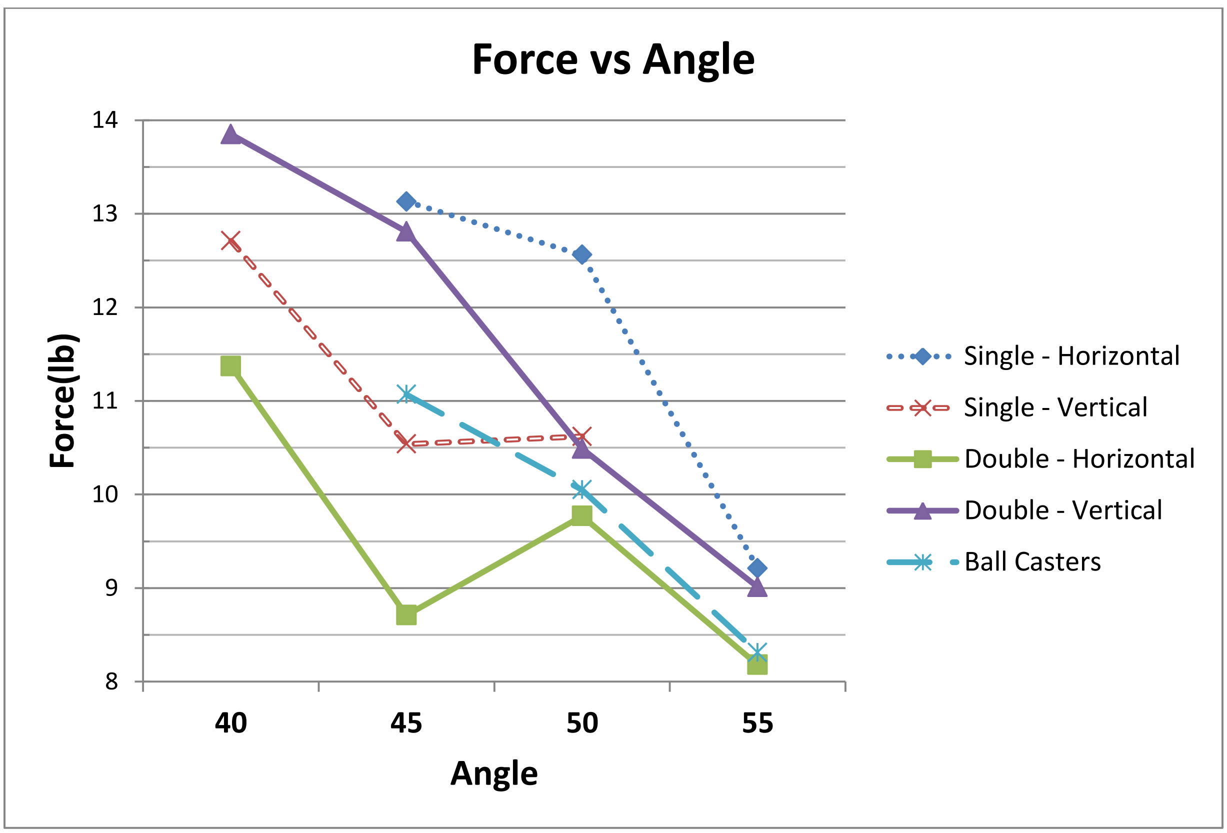
Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
45	1	1.013	7.0099E-01	5.8682	12.9371	26.6	58.6	0.2206
	2	0.688	7.1411E-01	4.4128	9.7285	26.6	58.6	0.1659
	3	0.471	7.1075E-01	4.7855	10.5502	26.6	58.6	0.1799
	Average	-	0.708617	5.0222	11.0719	-	-	0.1888

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
50	1	0.4	7.1960E-01	3.8038	8.3859	26.6	58.6	0.1430
	2	1.342	6.9824E-01	6.1732	13.6096	26.6	58.6	0.2321
	3	0.996	7.2052E-01	3.7017	8.1609	26.6	58.6	0.1392
	Average	-	0.712787	4.5596	10.0521	-	-	0.1714

Angle	Run #	Time (s)	Voltage (V)	Force (kg)	Force (lb)	Weight (kg)	Weight (lb)	Friction Coefficient
55	1	0.521	7.1686E-01	4.1077	9.0560	26.6	58.6	0.1544
	2	0.521	7.2174E-01	3.5664	7.8625	26.6	58.6	0.1341
	3	0.938	7.2113E-01	3.6340	8.0117	26.6	58.6	0.1366
	Average	-	0.719910	3.7694	8.3101	-	-	0.1417

Appendix C: Average Data

Angle	Measurement	Single - Horizontal	Single - Vertical	Double - Horizontal	Double - Vertical	Ball Casters
40	Force (lb) -->		12.71	11.37	13.85	
	Friction -->		0.212	0.1855	0.2261	
45	Force (lb) -->	13.13	10.54	8.71	12.81	11.07
	Friction -->	2.191	0.1759	0.1421	0.2091	0.1888
50	Force (lb) -->	12.56	10.62	9.77	10.49	10.05
	Friction -->	0.2095	0.1772	0.1595	0.1713	0.1714
55	Force (lb) -->	9.21		8.18	9.01	8.31
	Friction -->	0.1535		0.1336	0.147	0.1417



Appendix D: Minitab Analysis

SINGLES:

One-way ANOVA - Vertical: V40, V45, V50

```
Source  DF        SS        MS     F      P
Factor   2  0.002518  0.001259  2.24  0.188
Error    6  0.003378  0.000563
Total    8  0.005896

S = 0.02373   R-Sq = 42.71%   R-Sq(adj) = 23.61%


                            Individual 95% CIs For Mean Based on
                            Pooled StDev
Level  N     Mean    StDev  ---+---------+---------+---------+------
40     3  0.21200  0.01684              (-----------*----------)
45     3  0.17591  0.02171  (-----------*----------)
50     3  0.17716  0.03056   (----------*----------)
                            ---+---------+---------+---------+------
                             0.150     0.180     0.210     0.240

Pooled StDev = 0.02373


Grouping Information Using Tukey Method

    N     Mean  Grouping
40  3  0.21200  A
50  3  0.17716  A
45  3  0.17591  A

Means that do not share a letter are significantly different.


Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons

Individual confidence level = 97.80%


40 subtracted from:

      Lower    Center    Upper  ----+---------+---------+---------+-----
45  -0.09555  -0.03609  0.02336  (--------------*--------------)
50  -0.09430  -0.03484  0.02461  (--------------*--------------)
                                ----+---------+---------+---------+-----
                                 -0.080    -0.040     0.000     0.040


45 subtracted from:

      Lower   Center    Upper  ----+---------+---------+---------+-----
50  -0.05821  0.00125  0.06071          (--------------*--------------)
                               ----+---------+---------+---------+-----
                                -0.080    -0.040     0.000     0.040
```

One-way ANOVA - Horizontal: H45, H50, H55

```
Source  DF        SS        MS     F      P
Factor   2  0.007522  0.003761  7.74  0.022
Error    6  0.002915  0.000486
Total    8  0.010437

S = 0.02204   R-Sq = 72.07%   R-Sq(adj) = 62.76%


                            Individual 95% CIs For Mean Based on
                            Pooled StDev
Level  N     Mean    StDev  -----+---------+---------+---------+----
H45    3  0.21906  0.01914                     (--------*-------)
H50    3  0.20952  0.01634                  (--------*--------)
H55    3  0.15352  0.02871  (--------*--------)
                            -----+---------+---------+---------+----
                               0.140     0.175     0.210     0.245

Pooled StDev = 0.02204


Grouping Information Using Tukey Method

     N     Mean  Grouping
H45  3  0.21906  A
H50  3  0.20952  A
H55  3  0.15352    B

Means that do not share a letter are significantly different.


Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons

Individual confidence level = 97.80%


H45 subtracted from:

        Lower    Center     Upper     +---------+---------+---------+---------
H50  -0.06477  -0.00954   0.04569              (--------*---------)H55  -0.12077  -
0.06554  -0.01031     (--------*--------)
                                      +---------+---------+---------+---------
                                   -0.120    -0.060     0.000     0.060


H50 subtracted from:

        Lower    Center     Upper     +---------+---------+---------+---------
H55  -0.11122  -0.05600  -0.00077      (---------*--------)
                                      +---------+---------+---------+---------
                                   -0.120    -0.060     0.000     0.060
```

Two-Sample T-Test and CI: V, H

```
Two-sample T for V vs H

   N    Mean   StDev  SE Mean
V  9  0.1884  0.0271   0.0090
H  9  0.1940  0.0361    0.012


Difference = mu (V) - mu (H)
Estimate for difference:  -0.0057
95% CI for difference:  (-0.0380, 0.0266)
T-Test of difference = 0 (vs not =): T-Value = -0.38  P-Value = 0.712  DF = 14
```

DOUBLES:

Multilevel Factorial Design

```
Factors:      2     Replicates:     3
Base runs:    8     Total runs:    24
Base blocks:  1     Total blocks:   1

Number of levels: 4, 2
```

General Linear Model: Friction versus Angle, Orientation

```
Factor       Type   Levels  Values
Angle        fixed       4  40, 45, 50, 55
Orientation  fixed       2  Horiz., Vert.


Analysis of Variance for Friction, using Adjusted SS for Tests

Source             DF     Seq SS     Adj SS     Adj MS      F      P
Angle               3  0.0132337  0.0132337  0.0044112  11.00  0.000
Orientation         1  0.0066035  0.0066035  0.0066035  16.47  0.001
Angle*Orientation   3  0.0030742  0.0030742  0.0010247   2.56  0.092
Error              16  0.0064162  0.0064162  0.0004010
Total              23  0.0293276


S = 0.0200253   R-Sq = 78.12%   R-Sq(adj) = 68.55%
```

CASTERS:

One-way ANOVA: 45, 50, 55

```
Source  DF       SS       MS     F      P
Factor   2  0.00340  0.00170  1.38  0.321
Error    6  0.00739  0.00123
Total    8  0.01079

S = 0.03509   R-Sq = 31.53%   R-Sq(adj) = 8.71%


                            Individual 95% CIs For Mean Based on
                            Pooled StDev
Level  N     Mean    StDev  -------+---------+---------+---------+--
45     3  0.18880  0.02842              (-----------*------------)
50     3  0.17141  0.05257         (------------*-----------)
55     3  0.14171  0.01109  (-----------*------------)
                            -------+---------+---------+---------+--
                                 0.120     0.160     0.200     0.240

Pooled StDev = 0.03509


Grouping Information Using Tukey Method

    N     Mean  Grouping
45  3  0.18880  A
50  3  0.17141  A
55  3  0.14171  A

Means that do not share a letter are significantly different.


Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons

Individual confidence level = 97.80%


45 subtracted from:

       Lower    Center    Upper  ---------+---------+---------+---------+
50  -0.10532  -0.01739  0.07054      (------------*-----------)
55  -0.13503  -0.04710  0.04084  (-----------*------------)
                                 ---------+---------+---------+---------+
                                       -0.070     0.000     0.070     0.140


50 subtracted from:

       Lower    Center    Upper  ---------+---------+---------+---------+
55  -0.11764  -0.02971  0.05823    (------------*-----------)
                                 ---------+---------+---------+---------+
                                       -0.070     0.000     0.070     0.140
```

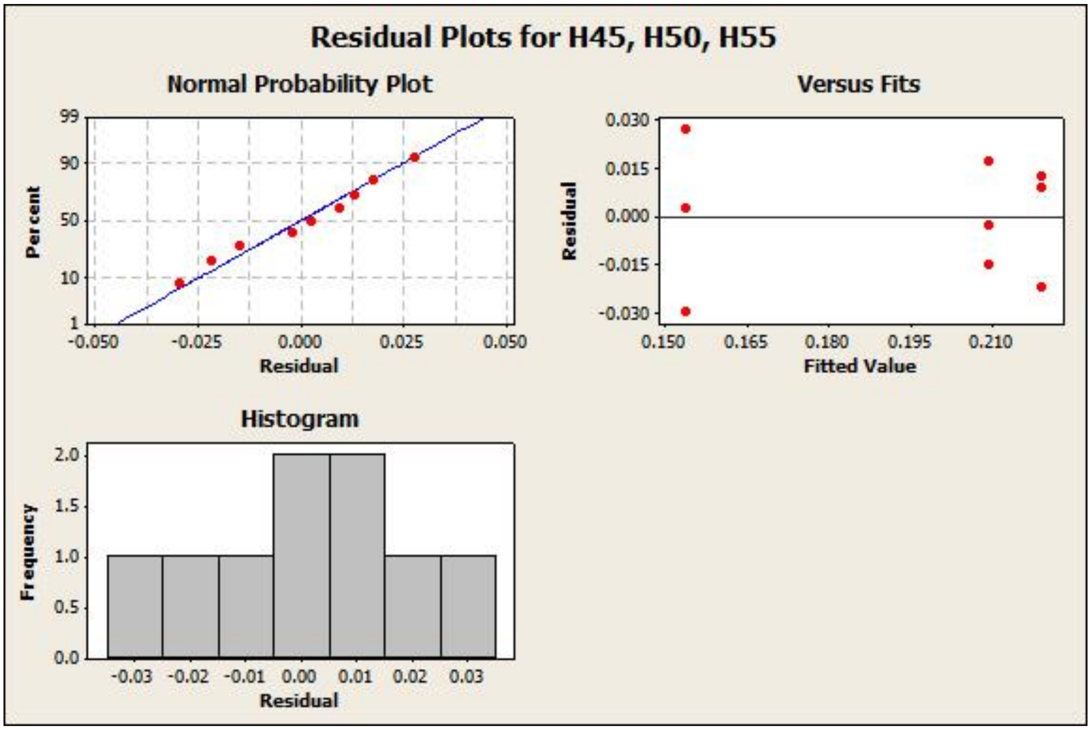
OVERALL:

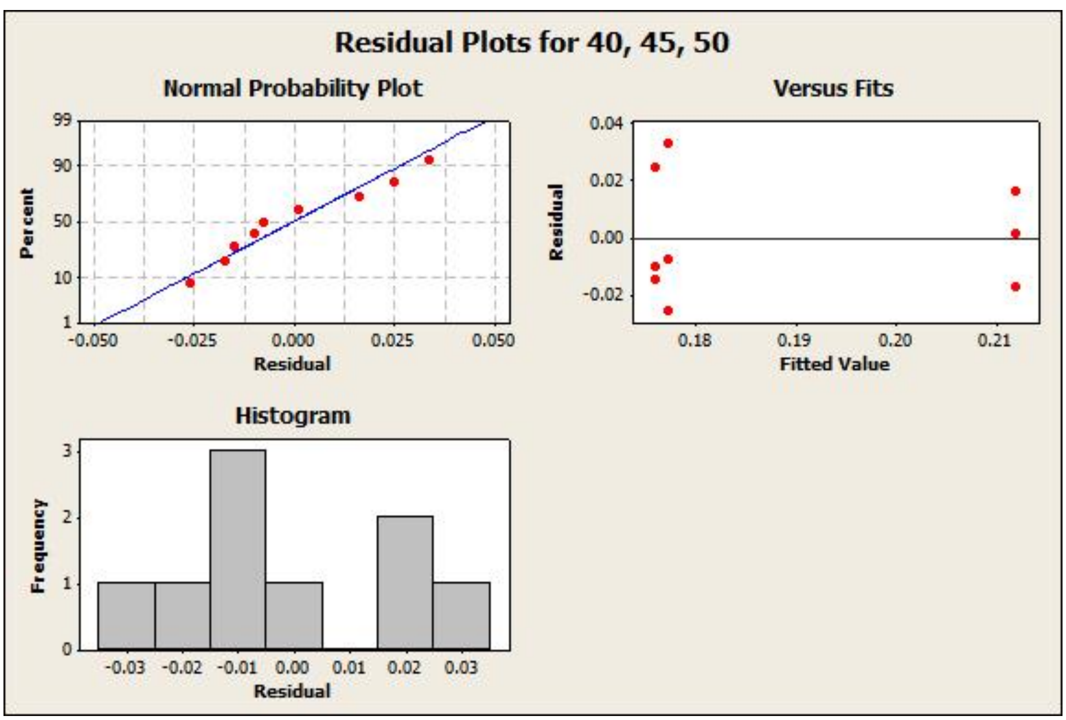
One-way ANOVA: Single, Double, Casters

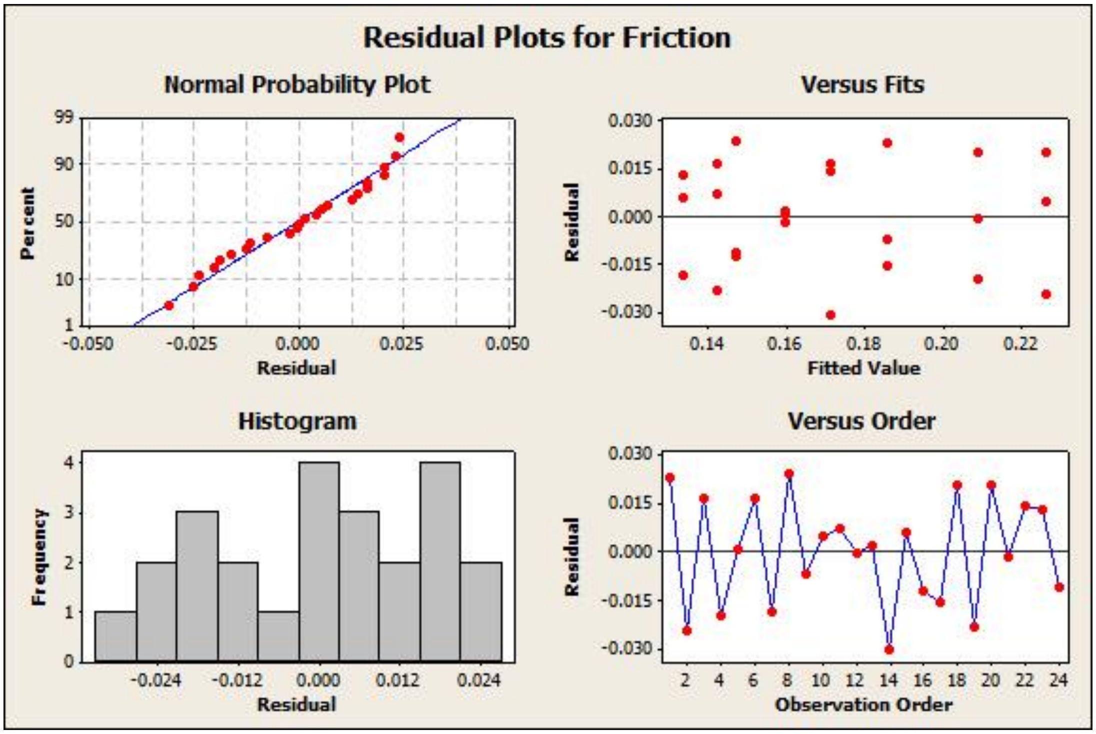
Source	DF	SS	MS	F	P
Factor	2	0.00509	0.00255	2.16	0.126
Error	48	0.05660	0.00118		
Total	50	0.06169			

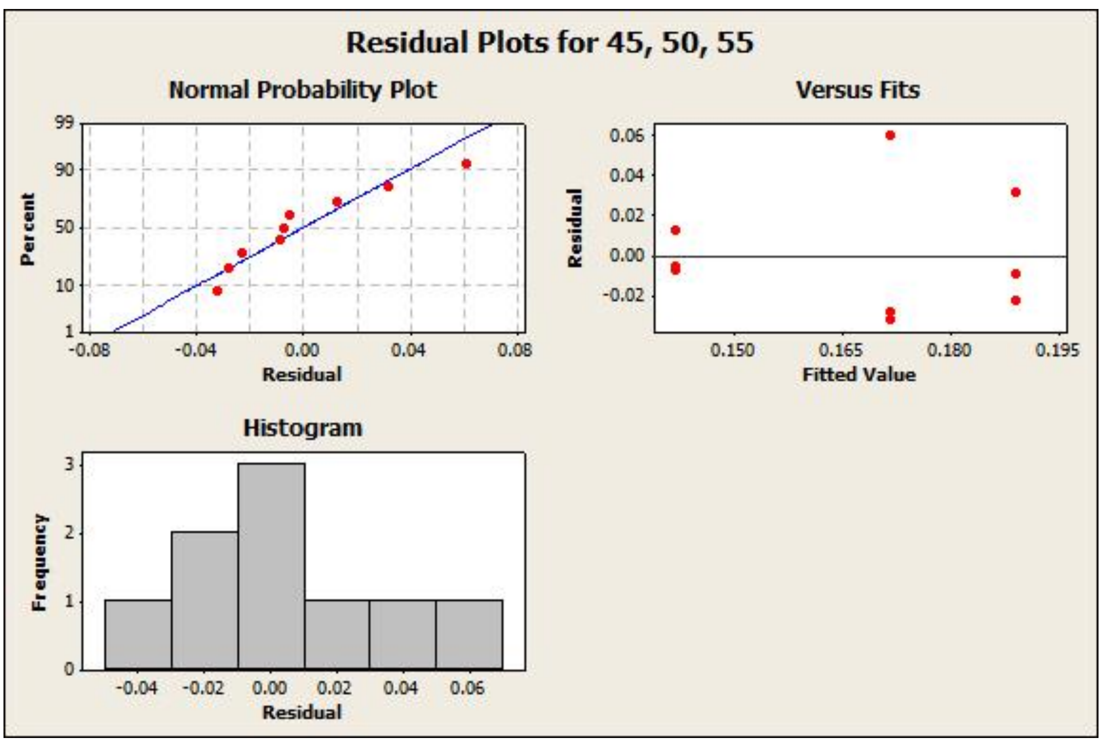
S = 0.03434   R-Sq = 8.26%   R-Sq(adj) = 4.43%

Appendix E: Minitab Residual Plots






Residual Plots for Friction
Normal Probability Plot
Percent
Residual
Versus Fits
Residual
Fitted Value
Histogram
Frequency
Residual
Versus Order
Residual
Observation Order


Residual Plots for 45, 50, 55
Normal Probability Plot
Percent
Residual
Versus Fits
Residual
Fitted Value
Histogram
Frequency
Residual