

Activity 19b: Logarithms

1. a) In the following, find the value of each expression using the information given. You don't need a calculator, but make sure to show your work.

i. What is $\log\left(\frac{a^2}{b}\right)$ when $\log a = 2$ and $\log b = -1$?

ii. What is $\log(\sqrt[3]{xy})$ when $\log x = 3$ and $\log y = -2$?

iii. What is $\ln(e^{\ln(ab)})$ when $\ln a = \ln b = 2$?

b) For the first two questions in part a, we didn't specify if the logarithms were in base 10, base 2, or something else. Does it matter? Why or why not?

2. Though seemingly abstract, logarithms are actually an intuitive tool for comparing things of very different sizes. This is embodied in the scientific jargon, "order of magnitude", which denotes a factor of 10 difference. For example, we might say that a billionaire (~\$1 billion) is roughly "three orders of magnitude richer" than a millionaire (~\$1 million), since:

$$\log_{10} \left(\frac{\$10^9}{\$10^6} \right) = \log_{10}(10^3) = 3.$$

a) In the following, figure out approximately how many orders of magnitude there are between different quantities. When not given, use whatever values seem reasonable to you, but make sure to write down your choice of values. Show your work.

i. Area of New Mexico (~300,000 km²) and area of the world (~500 million km²)

ii. Mass of a bicycle and mass of a car

iii. Radius of an apple and radius of an atom ($\sim 10^{-10}$ m)

b) Suppose that, for some quantities x , y , and z , x is 3 orders of magnitude larger than y and y is 4 orders of magnitude larger than z . How many times greater is x than z (i.e. what is $\frac{x}{z}$)? How many orders of magnitude difference is there between x and z ? Show your work below, making sure to clearly state which logarithm rules you are using.

3. A common tool used in engineering and the sciences is a semilog plot. In such a plot, the independent variable, x , is displayed on the horizontal axis, but the *logarithm of the dependent variable*, $\ln y$, is displayed on the vertical axis. Why would we do this, when we could just plot y as a function of x ? The main motivation has to do with conveniently plotting exponential data, as we'll see.

a) Suppose we obtain data in an experiment and plot it on a semilog plot. In many realistic situations, we'll obtain a graph that is very close to a straight line. In other words, we will find that for some m and b , it is true that $\ln y = mx + b$. In this case, what is $y(x)$? What *type* of function is this?

b) If there is data that we expect to match an exponential function, $y(x) = y_0 e^{x/x_0}$, describe below a way to figure out the parameters y_0 and x_0 by plotting our data on a semilog plot.

c) Another type of plot that is very useful in real-world applications is a log-log plot, where we plot $\ln y$ on the vertical axis and $\ln x$ on the horizontal axis. If our data makes a straight line on this type of plot, (in other words, $\ln y = m(\ln x) + b$, for some m and b), what is $y(x)$? What *type* of function is this?