

## Geometry and Astronomy

Geometry comes from the roots “geo” meaning Earth and “metron” meaning measurement. The ancient Greeks used it to measure the Earth, but also to figure out the distances to the Moon and Sun.

Firstly, even the ancients appreciated that the Earth is round. On a calm sea, when ships go over the horizon, the last part you see is the top of the mast... a clear indication that the Earth is curving downward.

1. Eratosthenes (276-194 BC) calculated the circumference of the Earth. He knew that, at noon on the summer solstice (the longest day of the year), the Sun is directly overhead in a town called Syene (now Aswan), in the south of Egypt. In his hometown of Alexandria, he measured the shadow cast by a perfectly vertical pole that was 10 cubits high at the same time, and found the shadow was one and a quarter cubits long. He also knew that that Syene was directly south of Alexandria and 5,000 stadia (about 500 geographical miles or 927.7 km) away. He *assumed* that the Sun was very far away.

What is the circumference and diameter of the Earth?

2. Size of & Distance to the moon. When the Sun casts a shadow of any object, there is a very dark part of the shadow, call the umbra. There is a lighter part of the shadow around that, called the penumbra. If you are looking from a point in the penumbra, you can see part (but not all) of the Sun; but in the umbra, you can see none of the Sun. Draw a sketch of the Sun and an spherical object that is perhaps one tenth as large. Draw lines from the top and bottom edges of the Sun so that you can find the region of the umbra.

Any object that is the same distance from the Sun will have the same ratio of its diameter to the length of its umbra. (You could measure this by looking at the shadow of a quarter, for example... of course, the ancient Greeks did not have quarters!) On Earth, this ratio is 108.

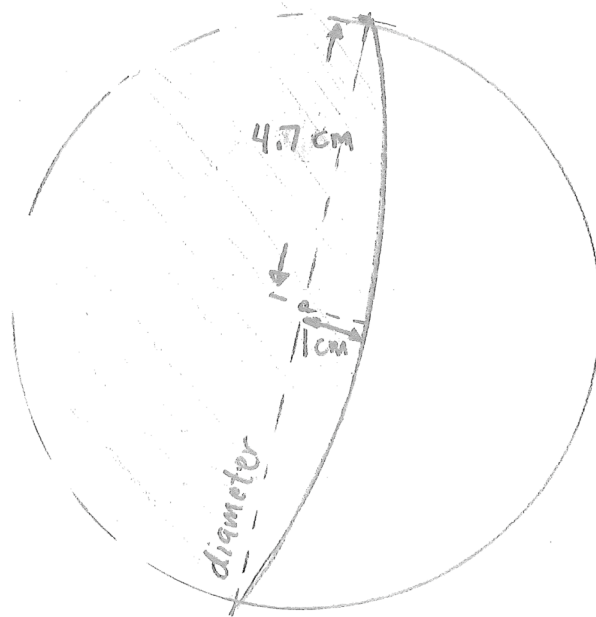
It just so happens that, during a “solar” eclipse, the moon is just large enough to block out the entire Sun. (If you see a total solar eclipse, are you in the umbra or the penumbra?) At any given instant, the area of the Earth that experiences a total eclipse of the Sun is quite small. In fact, in “annular” eclipses, there is no umbra on the Earth. So let’s take as a good approximation that the

umbra from the Moon just reaches the Earth. What is the ratio of the distance to the moon to its diameter?

During a lunar eclipse, the (spherical) Earth casts a shadow on the moon. The shadow of the earth (at the distance of the moon) is bigger than the moon. Here's a sequence of three pictures taken during a lunar eclipse.



Here's a geometric representation of the middle picture. Use this picture to calculate the size of the Earth's shadow, in terms of moon diameters.



Of course, just like everything else, the total length of the Earth's shadow is 108 Earth diameters.

Now for the hardest part. What is the length of the Earth's shadow, in terms of moon orbit radius? Hint: consider the triangle formed by the moon's shadow, and the triangle formed by the part of Earth's shadow that is farther than the moon? Are these similar triangles?

Now that you have the total length of the Earth's shadow in terms of Earth diameters (which you know from part 1) and in terms of moon orbit radius, you can calculate the moon orbit radius. Do so. While you're at it, calculate the diameter of the moon.

3. Distance to the Sun. Aristarchus, a colleague of Eratosthenes, measured the distance to the Sun, using the known distance to the moon. His brilliant insight is that when there is an exact half moon, the angle Sun-Moon-Earth is exactly  $90^\circ$ . He measured the angle Sun-Earth-Moon and found it to be  $87^\circ$  during a perfect half-moon (the correct angle is actually much closer to  $89.9^\circ$ !) What did Aristarchus report for the distance to the Sun? What did he report for its diameter?

4a. The solar power received by the Earth when the Sun is directly overhead is 1 kilowatt per square meter, or 1000 Joules per square meter per second. Using your result from 3, estimate how much energy the Sun emits every second.

4b. If you burn gasoline, you can get 42,000 Joules per gram. Assume the density of gasoline is about 1 gram per  $\text{cm}^3$ , and calculate how long the Sun could burn if it were a giant ball of gasoline!

[A historical note: William Thompson (Lord Kelvin) made this calculation around 1860. He used it to argue that Charles Darwin was wrong about evolution, as the Sun could not possibly have been around long enough! Hans Bethe only figured out how the Sun can burn so bright for so long nearly 100 years later.]