

Activity 20, 21: Determining the Yield of the World's First Atomic Bomb

Adapted from a write-up by Gary Condon '97, grad student at UNM

Today's activity is a bit different. You'll be doing an interesting "experiment", while reviewing several of the math skills that you've covered so far.

Background: Shortly before dawn on July 16, 1945, the United States Army detonated the world's first atomic bomb, codenamed "Trinity", near Alamogordo in southern New Mexico. By compressing a sphere of plutonium that could fit in the palm of your hand to less than half of its original size, the bomb initiated a runaway chain reaction of nuclear disintegration by which a massive amount of energy was liberated in only a tiny fraction of a second. The ensuing blast could be seen as far away as Santa Fe (160 miles) and could be heard up to 100 miles away. Although many civilians witnessed the explosion, the nature of this new weapon would not be disclosed to the public until three weeks later, on August 6th, when a uranium bomb of a different design, was dropped on the Japanese city of Hiroshima. On August 15th, World War II ended when Emperor Hirohito of Japan officially accepted the terms of surrender six days after the destruction of the city of Nagasaki by a second, "Trinity"-type plutonium bomb.

In this activity, you will make measurements on photographs of the explosion of the Trinity device and then use your result to determine the explosive energy of the bomb. The techniques described in this activity were actually used in the determination of the bomb's effects after the test. It is important to note that

theoretically estimating the yield of the bomb was so difficult that even the project scientists, many of them Nobel laureates, gave predictions ranging from a complete dud to the total destruction of the Earth.

Experiment:

1. Dimensional Analysis.

The primary factors influencing the expansion of an explosion are the energy released in the explosion, i.e., the yield (E), the mass density of the air surrounding the explosion (ρ), and the time elapsed since the detonation of the explosive device (t). Assume that the radius (R) of the explosion can be written in the following form

$$R = E^A \rho^B t^C$$

where the exponents A , B and C are to be determined. An equation like this would generally have a constant of proportionality as a multiplier. It was found experimentally, that if E , ρ , and t are in SI units, this constant has a value of approximately 1. Thus, for our analysis, we will leave it out of the equation.

Before doing this, predict whether each of the exponents A, B, and C should be positive or negative. For example, one would expect that the radius would increase with time as the explosive material expands. What does this say about “C”, i.e., should “C” be positive or negative?

C is _____

Likewise, make predictions of the signs of “A” and “B”:

A is _____

B is _____

To determine possible values for the exponents A, B, and C, we will make use of the fact that both sides of the equation $R = E^A \rho^B t^C$ must be dimensionally consistent, i.e., the units on both sides must match. In terms of the fundamental units of mass (M), length (L) and time (T), the units of energy are $(M L^2 T^{-2})$, the units of mass density are $(M L^{-3})$ and the units of time and radius are (T) and (L) respectively. Plugging these units into our general equation above gives the following “dimensional” equation

$$(L) = (M L^2 T^{-2})^A (M L^{-3})^B (T)^C$$

For each fundamental unit (M, L and T), one can write down an equation matching the powers of that unit on both sides of the equation. For example, if the units of length are to be balanced on both sides of our dimensional equation, the following equation must be satisfied

$$1 = 2A - 3B \text{ (since } L^1 = L^{2A} L^{-3B} = L^{2A-3B}\text{)}$$

In the space provided below, write down the additional two equations that must be satisfied for the units of mass (M) and time (T) to balance on both sides of our dimensional equation.

It should be noted that while dimensional analysis does not guarantee that an equation fits the physical phenomena, it is a necessary condition for the equation to be correct.

2. Simultaneous Equations.

Since there are three simultaneous equations and three unknowns, the exponents (A, B and C) can be uniquely determined. Determine the values of A, B and C and enter them in the spaces provided below. Make sure to show your steps below!

A= _____ B = _____ C = _____ *Check if your previously predicted signs for A, B and C agree with the math.*

With the determination of the exponents, the form of our assumed equation for the expansion of the explosive shockwave is complete and ready for comparison with actual data. The remainder of this report deals with the extraction of data from photographic records and their comparison with the simple model derived in this section.

3. Analysis of Photographic Records.

In the appendix at the end, you will find copies of real photos taken from the Trinity test site. The photos were taken at 4000 or 8000 frame/sec. Choose 8 of the best quality (sharp, well-defined spherical image) still frames of the explosion from photo archive #1 and also use photo archives # 2 and #3. The photos are time-stamped and have scale bars superimposed on the images. A blank table for recording data is provided below. Begin by entering the time “t” for your chosen images in column 2, making sure to convert “ms” to “s”. The steps for filling out the remainder of the table follow after the table.

Photo #	Time, t (s)	Scale Factor	Apparent Diameter (cm)	True Radius, R (m)	x = log(t)	y = log(R)
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$$1 \quad \begin{array}{l} 100\text{m} \\ : \\ \text{---} \\ \text{m} \end{array} \text{c}$$

$$2 \quad \begin{array}{l} 0.01 \\ 5 \end{array} \begin{array}{l} 100\text{m} \\ : \\ \text{---} \\ \text{m} \end{array} \text{c}$$

$$3 \quad \begin{array}{l} 0.12 \\ 7 \end{array} \begin{array}{l} 100\text{m} \\ : \\ \text{---} \\ \text{m} \end{array} \text{c}$$

Next, determine the scale factors of the photo archives #1, #2, and #3. Measure the length of the “100 m” line that is superimposed over each archive. Using a ruler, record to the nearest 0.01 cm and record in column 3.

In column 4, record the apparent diameter of the fireball to the nearest 0.01 cm, as measured from the photographs. Measure

horizontally across the widest part of the fireball.

Using columns 3 and 4, determine the real radius of the fireball “R”, in meters, paying attention to the correct number of significant figures. Record in column 5.

Make the calculations indicated in columns 6 and 7. The reasons for these calculations will become apparent in the next section.

4. Comparison of Photographic Records to Theory.

Note that our assumed equation, $R = E^A \rho^B t^C$, is not linear. But with a math trick, you can turn it in to a linear equation of sorts. It’s likely that you have not seen this before, so make sure to follow steps and you’ll see the magic unfold.

Take the logarithm of both sides of $R = E^A \rho^B t^C$. Show that this can be written as $\log(R) = C \log(t) + \log(E^A \rho^B)$.

If we let $y = \log(R)$ and $x = \log(t)$, then the equation above has the form of an equation for a straight line, $y = mx + b$. Then, comparing the two equations $\log(R) = C \log(t) + \log(E^A \rho^B)$ and $y = mx + b$ what must the slope “m” and the y-intercept “b” equal

in terms of E, ρ , A, B, and C?

m = _____

b = _____

Since the equation $\log(R) = C \log(t) + \log(E^A \rho^B)$ is linear, then a plot of $\log(R)$ versus $\log(t)$ should fall on a straight line.

A comparison of the slope from such a graph with the value of "C" derived in part 2 will constitute a test of the theory. If the theory stands up to such a test, then the yield of the bomb can be extracted from a measurement of the y-intercept of the graph.

Using the Logger Pro software, plot $y = \log(R)$ versus $x = \log(t)$. Be sure to clearly label both the axes of the graph; your instructor will show you some specifics, but here are a few pointers for Logger Pro. First, spread out your data by selecting the "Autoscale" icon. Then, to see the individual data points, rather than a connect-the-dots type graph, choose "Options" and then "Graph Options". Next, toggle "Connect Points". By doing this, you will see the individual points. Print your graph to turn in. Using a ruler, lightly superimpose a linear "best-fit" over the individual data points. If any of them are far off from being linear, they can be omitted. Now find "m" and "b". Do this by selecting the "Curve Fit" button, then toggle "Linear" and select "Try Fit". Record the slope and y-intercept:

m = _____

b = _____

As a test of the theory, compare the theoretical value of C (part 2) with your experimental value of C (part 4) by computing a percent error:

$$\%error = \frac{|\text{value of } C \text{ (part 2)} - \text{value of } C \text{ (part 4)}|}{\text{value of } C \text{ (part 2)}} \times 100\% \approx \underline{\hspace{2cm}}$$

Is this error larger or smaller than you expected? Comment on your reasons.

Using the y-intercept that you found on the previous page, algebraically, solve for E in terms of b, ρ , A, and B. Show your steps below.

Re-write the numerical values that you found for b , A , and B from previous pages.

$b =$ _____ $A =$ _____ B
 $=$ _____

Convert ρ to SI units, kg/m^3 ; the density of dry air at the test site is 0.992 g/l . Note that $1 \text{ l} = 1000 \text{ cm}^3$.

To find the value of E in SI units*, substitute your value of b , ρ , A , and B into your equation. Show your steps below.

explosive yield: $E = \underline{\hspace{2cm}}$ (kg m²/s²)

*When doing certain mathematical operations, e.g., taking logarithms, the treatment of units can be tricky. One can only take logarithms of pure numbers, but not the units. Therefore, it is imperative that the units match on both sides of an equation before taking logarithms. Only when the units are consistent will logarithms yield valid mathematical relations. In our case, that means using SI units throughout.

Given that the explosive yield of a single ton of TNT is 4.2×10^9 kg m²/s², convert your result above into an equivalent yield in thousands of tons (kilotons) of TNT .

explosive yield: $E = \underline{\hspace{2cm}}$ (kilotons of TNT)

The “official” yield was 18.6 kilotons of TNT. Comment on any reasons for differences between your experimental value of E versus the “official” value of E.