

Activity 4: Proportions and Ratios

1. a) An old, rusty bathroom scale at your friend's house reads 72 kg when you step on it. Your scale at home (which you know is accurate) reads 140 lbs. Write two proportionality equations, the first giving the weight reading on your friend's scale (in kg) as a function of the reading on your scale (in lbs), and the second giving your friend's scale's reading as a function of your actual weight in kg. What are the units of the proportionality constants in each equation?

b) While holding your friend's cat, you step on his old scale again and see the reading at 77 kg. What is the actual weight of your friend's cat, in pounds? Give the math supporting your answer below.

2. Newton's law of cooling says that objects at a certain temperature are cooled by convective heat transfer* at a rate that is proportional to the surface area of the object. In equations, if P is the rate at which an object loses heat to the environment, and A is its surface area, $P = cA$, where c depends on many factors, such as the temperature of the object and environment, the rate of fluid flow, the type of fluid, etc.

a) An engine you are designing, the Mark 1, will be cooled by a fan which blows air over it. If the Mark 1 is shaped roughly like a cube with side length of one meter, and it releases 130 MJ** of heat every hour, what does c have to be to keep the engine at a constant temperature (i.e. heat generated = heat dissipated by cooling)? Express c in units of kilowatts per square meter.

b) Your boss assigns you to design the Mark 2, which is similar to the Mark 1, but has 6 times the fuel consumption (and therefore heat generation). By what factor does the side length of your cube-shaped engine need to increase so that it can keep the same value of c (which is related to, among other things, the temperature of the engine) as the Mark 1?

*Convective heat transfer is heat transfer caused by moving fluids (such as air or water) in contact with an object. For example, blowing on a hot spoonful of soup is an effective means of convectively cooling it so that you don't burn yourself!

** Some useful unit facts: a Joule (J) is the SI unit for energy, and a Watt (W) is the SI unit for power. 1 Watt equals 1 Joule per second.

c) In working on this redesign, you realize that the Mark 1 might have been improved by shaping it like a *sphere*. This hypothetical engine, which you dub the Mark 1X, would have produced the same amount of heat as the Mark 1, and had a radius of half a meter. Assuming the Mark 1X maintains a constant temperature, what is its c value?

d) If you wanted to design the Mark 2X, which has 6 times the heat generation of the Mark 1X (but the same c value), by what factor would you need to increase the radius of your sphere-shaped engine by? You should obtain the same answer as in part b, despite the different shapes of the Mark 1 and Mark 1X. Give a simple explanation for why this is.

3. During one rather dull afternoon, you decide to tear into your mother's spring collection and characterize the behavior of every spring she owns. You take a large amount of data, of which the following is fairly representative:

Spring Displacement (cm)	- 2.0	- 1.0	0	1.0	2.0	3.0	4.0	5.0
Spring Force (N)	- 9.5	- 5.3	0	5.2	10.3	14.9	16.5	17.4

(The negative numbers in the data above might look strange, but they only represent the direction of a displacement or force.)

a) Plot the above points on graph paper, with spring displacement on the x axis, and draw a curve that passes through all the data points. In your opinion, does this curve look like a straight line?

b) It is often the case that physical laws are only accurate descriptions of some parts of a system's behavior. In the case of springs, Hooke's law (the fact that the force of a spring is proportional to its displacement) is only valid when the spring isn't stretched out **too far**. With this in mind, ignore the data points on your graph that look like they don't lie on a straight line (the outliers), and draw a straight line that goes through the remaining data points. Calculate the slope of this line below, and give its value with the appropriate units. This quantity is traditionally called k , the spring constant of the spring.

c) The spring constant has the physical interpretation of how stiff or loose our spring is, with larger spring constants leading to stiffer springs. The spring constant also has the mathematical interpretation as the slope of a force vs. position graph. Using these two facts, say whether our spring is getting stiffer or looser as we stretch it out further and further. You should support your answer with evidence based on the shape of the force vs. displacement graph.