

General Power Laws

1. The Earth exerts a gravitational pull on any object, given by Isaac Newton's formula:

$$F = \frac{GMm}{r^2}$$

where G is the "universal gravitational constant", M is the mass of the earth, m is the mass of the object, and r is the distance of the satellite from the *center of the earth*.

The radius of the earth is 6400 km (at sea level).

- a. Mount Everest is almost 9 km above sea level. Estimate your weight on Mount Everest. (Your weight is the force with which gravity pulls you down, if you are not moving.)
- b. The radius of the moon is 27% of that of the earth. Your weight on the moon is $1/6^{\text{th}}$ your weight on Earth. Assuming equal densities for the moon and the earth, what is the ratio of the moon mass to the Earth's mass? (Note: the M in the equation is the mass of the moon, for a satellite in orbit around the moon.)
- c. A satellite orbiting the Earth may be said to be "weightless", but gravity still pulls on it. (It's weightless in the same way that objects in a falling elevator are weightless... until they hit the bottom! The satellite keeps falling but never hits.) At what height would the gravitational force on a satellite be half of the force on the ground?

2. An unbalanced force causes an object to accelerate (change speed **or** direction.) The gravitational force on a satellite causes it to constantly change direction, so that it moves in a circle. (Some satellites can have elliptical orbits, but we will concentrate on circular orbits here.) The acceleration of a satellite in a circular orbit is given by

$$a = \frac{mv^2}{r}, \text{ where } v \text{ is the speed of the satellite.}$$

Use this formula, along with $F = ma$ and the law of gravitation, to solve for the speed v in terms of the other quantities. (Notice that the mass of the satellite cancels out! What do you think this means?)

- a. What is the ratio of the orbital speeds for a satellite in a low Earth orbit and a satellite in a low moon orbit?

b. It takes about 90 minutes for a low earth orbit satellite to go around the earth. About how long does it take a low moon orbit satellite to go around the moon?

3. The orbital period, T , is the time to complete one orbit. It's equal to the distance traveled around the circle, divided by the speed. Solve for the orbital period of a satellite in a circular orbit, as a function of G , M , r . (Make sure there's no v in your expression.)

Your formula might be a little neater if you find the period squared. (That will get rid of square root signs.) You have just derived one of Kepler's three laws of planetary motion!

a. It takes the moon one month (well, more like 28 days) to go around the Earth. Find the distance to the moon, by comparing it to a low Earth orbit satellite.

b. How far away would a satellite of the Earth have to be to go around the Earth once per year? Could we put such a satellite in front of the Sun and use it to combat global warming? What technical issues might prevent this?