
Discrete Random Signal Processing and Filtering Primer with MATLAB Chapter One Worksheet

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Problem 1.3.1

Find the Fourier Transform of the comb function:

$$\text{comb}_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

The Fourier series representation of $\text{comb}_T(t)$ is:

$$f(x) = \sum_{n=-\infty}^{\infty} A_n e^{j2\pi nx/L}$$

$$A_n = \frac{1}{L} \int_{-L/2}^{L/2} f(x) e^{-j2\pi nx/L} dx$$

Hence for $\text{comb}_T(t)$

$$A_n = \frac{1}{T}$$

$$\text{comb}_T(t) = f(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} e^{j2\pi nt/T}$$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

$$\mathcal{F}\{f(t)\} = \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n/T)$$

Problem 1.4.1

Prove in the limit $T \rightarrow 0$ that

$$F(\omega) = T \sum_{n=-\infty}^{\infty} f(nT)e^{-j\omega nT}$$

$$F(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} \int_{(n-1)T}^{nT} f(t)e^{-j\omega t} dt$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} f(nT)e^{-j\omega nT} T$$

$$F(\omega) = T \sum_{n=-\infty}^{\infty} f(nT)e^{-j\omega nT}$$

Problem 1.5.1

Prove

$$\mathcal{F}_D \left\{ \frac{1}{N} F(n) \right\} = f(-k)$$

$$\mathcal{F}_D \{f(nT)\} = T \sum_{n=0}^{N-1} f(nT)e^{-j2\pi knT/NT}$$

$$\mathcal{F}_D \left\{ \frac{1}{N} F(n) \right\} = \frac{T}{N} \sum_{n=0}^{N-1} F(n)e^{-j2\pi knT/NT}$$

$$\mathcal{F}_D \left\{ \frac{1}{N} F(n) \right\} = T^2 \frac{1}{NT} \sum_{k=0}^{N-1} F(k) e^{j2\pi k(-n)T/NT} = T^2 f(-n)$$

Problem 1.5.2

Prove

$$\mathcal{F}_D \{ f(n - m) \} = F(k) e^{-j2\pi mk/N}$$

$$\mathcal{F}_D \{ f(nT) \} = T \sum_{n=0}^{N-1} f(nT) e^{-j2\pi knT/NT}$$

$$\mathcal{F}_D \{ f(n - m) \} = T \sum_{n=0}^{N-1} f(n - m) e^{-j2\pi knT/NT}$$

$$\mathcal{F}_D \{ f(n - m) \} = T \sum_{n=0}^{N-1} f(n) e^{-j2\pi k(n+m)T/NT} = F(k) e^{-j2\pi km/N}$$

Problem 1.5.3

Prove

$$f(n) e^{j2\pi nm/N} = \mathcal{F}_D^{-1} \{ F(k - m) \}$$

$$f(n) e^{j2\pi nm/N} = e^{j2\pi nm} \mathcal{F}_D^{-1} \{ F(k) \}$$

$$f(n) e^{j2\pi nm/N} = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{j2\pi kn/N} e^{j2\pi nm/N}$$

$$f(n) e^{j2\pi nm/N} = \frac{1}{N} \sum_{k=0}^{N-1} F(k - m) e^{j2\pi kn/N} = \mathcal{F}_D^{-1} \{ F(k - m) \}$$

Problem 1.5.4

Prove

$$Y(k) = \mathcal{F}_D \{ f(n) \otimes h(n) \} = F(k) H(k)$$

$$Y(k) = \mathcal{F}_D \{f(n) \otimes h(n)\} = \mathcal{F}_D \left\{ \sum_{m=-\infty}^{\infty} f(m)h(n-m) \right\}$$

$$Y(k) = \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} f(m)h(n-m)e^{-j2\pi nk/N}$$

$$Y(k) = \sum_{m=-\infty}^{\infty} f(m) \sum_{n=0}^{N-1} h(n)e^{-j2\pi nk/N} e^{-j2\pi mk/N}$$

$$Y(k) = \sum_{m=-\infty}^{\infty} f(m)e^{-j2\pi mk/N} H(k)$$

$$Y(k) = \sum_{m=0}^{N-1} f(m)e^{-j2\pi mk/N} H(k) = F(k)H(k)$$

Problem 1.5.5a

```
clear all

u = @(t) (1/2).*(1+sign(t));
f = @(t) exp(-t).*u(t);

T = 0.3;

y1 = f(0:T:0.6);
y2 = f(0:T:1.2);
y3 = f(0:T:20);

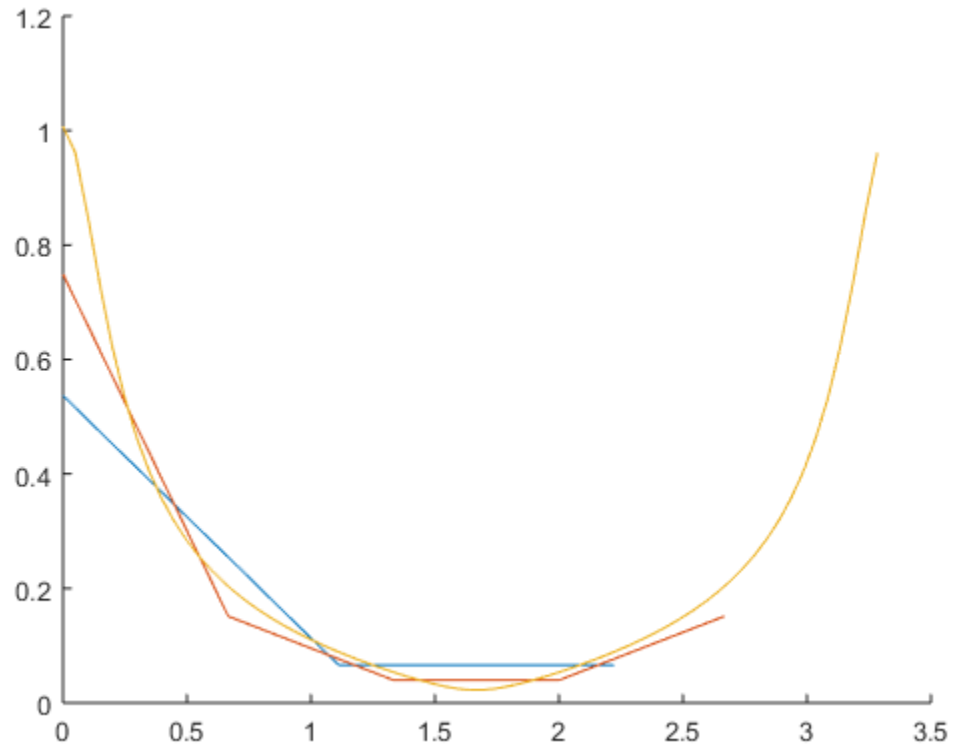
Y1 = T.*fft(y1);
Y2 = T.*fft(y2);
Y3 = T.*fft(y3);

clf
hold on
plot((0:length(Y1)-1)/(length(Y1)*T),abs(Y1))
plot((0:length(Y2)-1)/(length(Y2)*T),abs(Y2))
plot((0:length(Y3)-1)/(length(Y3)*T),abs(Y3))
hold off

abs([Y1(1) Y2(1) Y3(1)])

ans =
```

0.5369 0.7492 1.0075



Problem 1.5.5b

```
clear all

u = @(t) (1/2).*(1+sign(t));
f = @(t) exp(-t).*u(t);

T = 0.05;

y1 = f(0:T:0.6);
y2 = f(0:T:1.2);
y3 = f(0:T:2.0);

Y1 = T.*fft(y1);
Y2 = T.*fft(y2);
Y3 = T.*fft(y3);

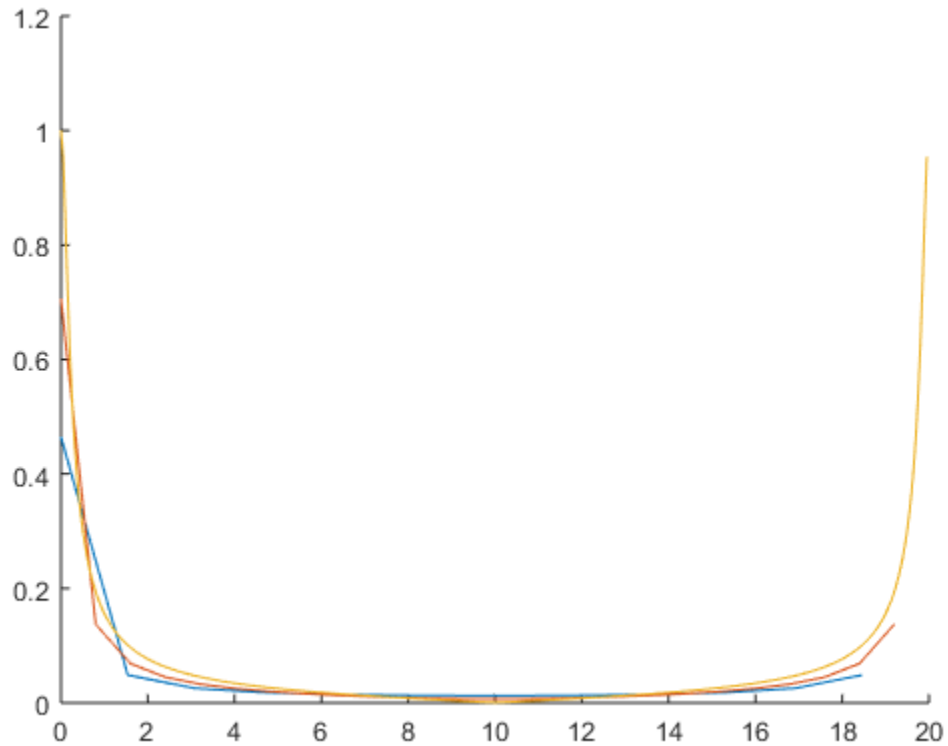
abs([Y1(1) Y2(1) Y3(1)])

clf
hold on
plot((0:length(Y1)-1)/(length(Y1)*T),abs(Y1))
```

```
plot((0:length(Y2)-1)/(length(Y2)*T),abs(Y2))  
plot((0:length(Y3)-1)/(length(Y3)*T),abs(Y3))  
hold off
```

```
ans =
```

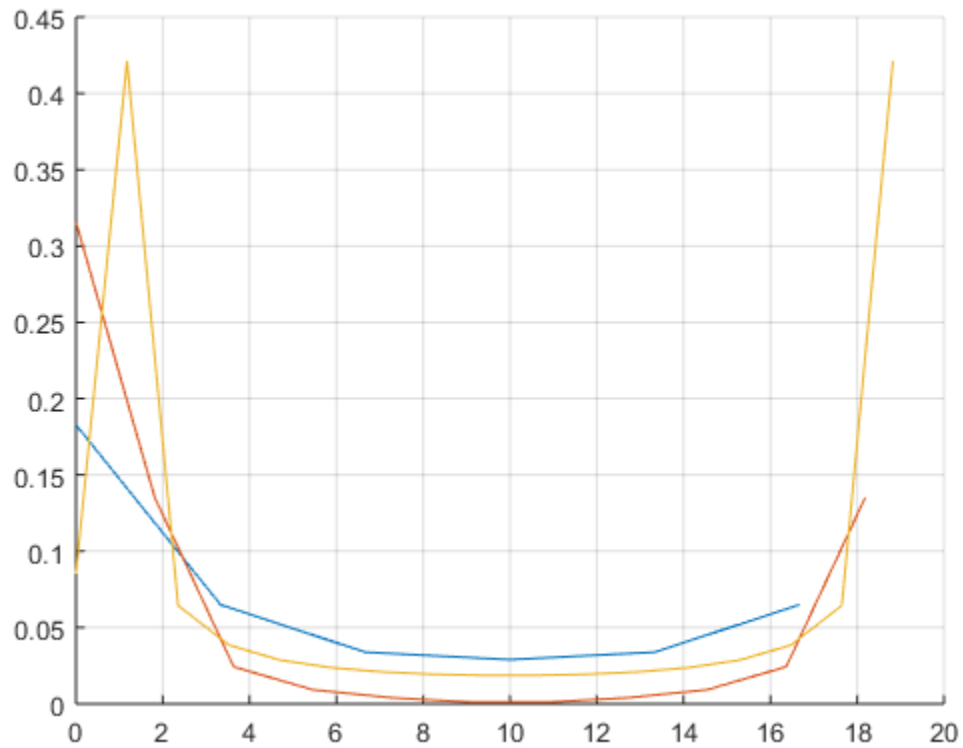
```
0.4650    0.7065    1.0002
```



Problem 1.5.6

```
clear all  
  
f = @(t) sin(2*pi*t);  
  
Period = 1;  
T = Period/20;  
  
y1 = f(0:T:(1/4)*Period);  
y2 = f(0:T:(1/2)*Period);  
y3 = f(0:T:0.8*Period);  
  
Y1 = T.*fft(y1);  
Y2 = T.*fft(y2);  
Y3 = T.*fft(y3);
```

```
clf
hold on
plot((0:length(Y1)-1)/(length(Y1)*T),abs(Y1))
plot((0:length(Y2)-1)/(length(Y2)*T),abs(Y2))
plot((0:length(Y3)-1)/(length(Y3)*T),abs(Y3))
hold off
grid on
```



Problem 1.5.7

```
clear all

x1 = [ ones(1,8) 0 0 0 ];
x2 = ones(1,11);

T = 1/sqrt(11);

X1 = T.*fft(x1);
X2 = T.*fft(x2);

[ sum(x1.*x1) sum(X1.*conj(X1)) sum(x2.*x2) sum(X2.*conj(X2)) ]

ans =
```

Problem 1.7.1

$$y(t) = \int_{-\infty}^{\infty} f(x)h(t-x)dx$$

$$y(t) = \int_{-\infty}^{\infty} e^{-x}u(x)e^{-2(t-x)}u(t-x)dx$$

$$y(t \geq 0) = \int_0^t e^{-x}e^{-2(t-x)}dx = e^{-t} - e^{-2t}$$

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